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THEORY AND PRACTICE IN DESIGNING

BY

HENRY ADAMS

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PREFACE

In the following pages the theory of structures has been dealt with from the most elementary examples to those which are more involved. The arrangement is progressive, so that the book may be used as a text-book by those beginning to study the subject, and they will be taken by easy steps right through. Only the simplest problems in mathematics are employed and the method of using all formulæ is clearly set out; graphic methods are employed whenever possible. The information when once fully explained is not repeated, so that the more complicated structures can be dealt with concisely, thus enabling the book to form a handy reference to those engineers and architects who are engaged in the practical design of structures. It is quite distinct from other books on designing by the same author and should be taken as complementary to them.

60, Queen Victoria Street, E.C.
Sept., 1911.

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THEORY AND PRACTICE IN DESIGNING

CHAPTER I

Timber, its Seasoning and Shrinkage—Principles of Stress

Timber is, perhaps, the best material to commence upon with a view to studying the stresses in structures and the principles of design. It is well known that wood consists of bundles of fibres interlaced so that they have small cohesion sideways, but considerable resistance to fracture across the fibres. Omitting slight irregularities of growth, the fibre, or grain, may be considered as always running lengthwise of the piece, and to be of the same character throughout.

In order to frame timber intelligently, it is necessary to know something about its behaviour in seasoning. Seasoning is the

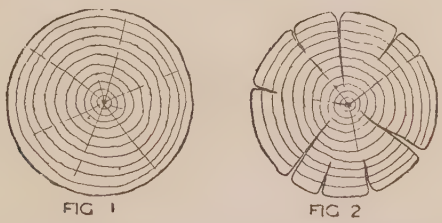


Fig. 1.—Section of Log showing Annual Rings.
Fig. 2.—Shrinkage of Log in Seasoning.

drying-out of moisture and sap after the timber is felled, and this is hastened by cutting it up into boards and scantlings. Fig. 1 shows the cross-section of a tree-trunk, with the annual rings of woody fibre or vascular tissue and medullary rays

composed of cellular tissue. If the log were left to season in this condition the evaporation of the moisture, which is distending the fibres, would cause the wood to shrink and split, or "shake," as shown in Fig. 2, and to prevent this it may be cut into planks, as shown in Fig. 3, to allow the air to act more freely upon it,

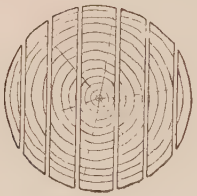


FIG 3

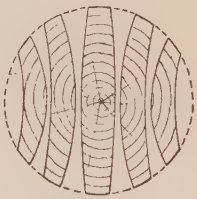


FIG 4

FIG. 3.—Log cut into Planks for Seasoning.
FIG. 4.—Shrinking and Warping of Planks.

and to give it greater freedom of movement. Omitting the outer slabs, which consist almost entirely of bark, the planks will shrink in seasoning as shown in Fig. 4. Upon studying this figure in connection with the rings and rays of Fig. 1, it will be seen that the shrinkage radially is so slight as to be negligible, and that the chief movement is circumferential; or, taking, say, a quarter of the tree and looking upon the medullary rays as the ribs of a lady's fan, the shrinkage takes place exactly as the fan would move in closing, the reason being that the rays are somewhat harder than the other part, and tend to retain their original dimensions. If the grain at the end of any piece of wood be carefully examined, the mode in which it will alter shape during any further drying, or conversely during any fresh access of moisture, can be confidently predicted. The shrinkage in length is very slight, but, owing to the shrinkage in width, a bevelled end is likely to alter considerably. The difference between a square end and a bevelled end in this respect is shown

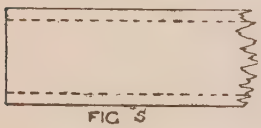


FIG 5

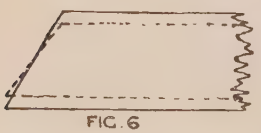


FIG. 6

FIG. 5.—Timber cut square shrinks square.
FIG. 6.—Timber cut on bevel alters angle in shrinking.

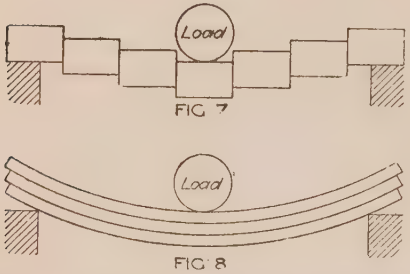
in Figs. 5 and 6, and the importance of using square ends whenever possible is self-evident.[1]

Wood is largely used for joinery and finishings in which questions of stress do not occur, but in carpentry, which has chiefly to do with structural work, and in which stability is a main consideration, it is necessary to consider the various kinds of stress and how they can best be met. At one time the terms "stress" and "strain" were used synonymously, but it is now customary to confine them each to its proper sphere. Stress is the internal resistance set up by the action of a load; strain is the alteration in shape produced by a stress, whether it be stretching, shortening, or bending. Thus we may say that a load of so many pounds on a beam produces a stress of so many pounds per square inch, and results in tensile and compressive strains, which permit a deflection bearing a certain ratio to the span.

The general principles of stress in beams can be easily followed if a rectangular beam of timber be considered. First, take a straight-grained piece of pine free from knots, square in section, of 1 in. side and 42 ins. long, let it be supported at the ends on rigid supports, 36 ins. apart, and hang a weight of 14 lbs. in the centre, which will produce a slight deflection or bending in the middle equal to, say, 0·15 in. Now consider what this bending means. It is evident that the length on the inside of the curve along the top cannot be the same as the length on the outside of the curve along the bottom, although both top and bottom were of the same length to start with. The alteration is, perhaps, too small for actual measurement, but the curvature would enable it to be calculated. However, that is a matter that can be left for the present. What is important is to notice that the load has produced a stress in the beam, resulting in a compressive strain in the upper part and a tensile strain in the lower part, both of them being greatest towards the centre. There is another strain which cannot be seen so readily, but nevertheless exists; it is due to a diagonal stress called shear, greatest towards the ends of the beam and least at the centre. The effects of shear may

[1] For full information upon this part of the subject, see "Joints in Woodwork" (E. and F. N. Spon, 6*d.*).

be indicated diagrammatically by Figs. 7 and 8, which indicate the tendency resolved into vertical and horizontal directions upon the principle of the parallelogram of forces (Fig. 9). Fig. 8 is very familiar in the woodyard or workshop when several match-



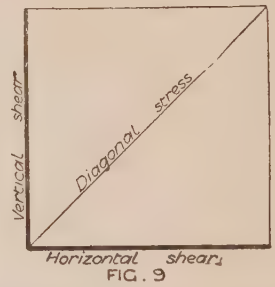


FIG. 7.—Illustration of Vertical Shear. FIG. 8.—Illustration of Horizontal Shear. FIG. 9.—Resolution of Diagonal Stress into Vertical and Horizontal Shear.

boards are laid on open bearings. The slipping between the surfaces is true horizontal shear, and in a solid piece of wood, although the actual sliding cannot take place, the tendency is still there. Fig. 10 shows the lines of stress in a beam and how they change from the horizontal direction in the centre to the diagonal direction towards the ends. They are really tensile and compressive throughout, but are called tension, compression, and shear for convenience. We shall return later on to the

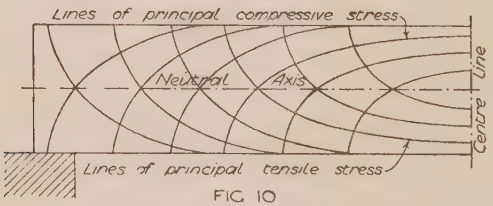


FIG. 10.—Lines of Stress in a Beam.

distribution of stress in the beam. At present it is desirable to assimilate general facts only.

The next point to consider will be the manner in which the material is disposed in the beam and the effect upon its strength. Suppose a unit beam, 1 in. square and 1 ft. span, loaded in the

centre until it breaks, this breaking load will amount to, say, 5 cwts. If the bulk of the beam be doubled by increasing its width, then exactly double the load, or 10 cwts., would be required to break it. If it were added to the length, by doubling the span, then only half the original load, or $2\frac{1}{2}$ cwts., would be required to cause fracture. If the extra material were added to the depth of the beam it would require a load of four times the original amount, or 20 cwts., to break the beam. Placing these results together for better comparison, thus :—

Breadth.	Depth.	Length.	Fracture Load.
1 in.	1 in.	12 ins.	5 cwts.
2 „	1 „	12 „	10 „
1 „	1 „	24 „	$2\frac{1}{2}$ „
1 „	2 „	12 „	20 „

It will be seen that the strength varies directly as the breadth, inversely as the length, and directly as the square of the depth. If ordinary fir had been taken, instead of specially selected material, the load to break the unit beam, given in the first line of the table, would have been probably $3\frac{1}{2}$ cwts.

The results of our investigations, so far, show that the strength of a fir beam may be indicated by the formula $W = \frac{cbd^2}{L}$, where W = ultimate breaking weight in cwts. in the centre, c = constant, or load to break unit beam = 3·5, b = breadth of beam in inches, d = depth of beam in inches, L = clear span of beam in feet. Example: Required, the breaking weight in the centre of a fir beam, 6 ins. broad, 12 ins. deep, and 15 ft. span.

$$W = \frac{cbd^2}{L} = \frac{3{\cdot}5 \times 6 \times 12^2}{15} = 201{\cdot}6 \text{ cwts.}$$

When the load is spread uniformly along the beam it requires exactly double the amount to produce the same result, or $201{\cdot}6 \times 2 = 403{\cdot}2$ cwts. will now be the breaking weight; but one does not want to break the beam except as a special test. In the ordinary way the beam is required to carry a load safely, and to do

this it is necessary to reduce the load to a fraction of the breaking weight, such as $\frac{1}{7}$, which is then called the factor of safety; or sometimes it is put the other way round, and the factor of safety is said to be 7. Thus the safe distributed load in cwts. on the beam would be $= \frac{cbd^2}{L} \times \frac{2}{7}$, or putting figures for the constant, $\frac{3{\cdot}5\ bd^2}{L} \times \frac{2}{7}$, but the figures cancel out, so that if we put W' for safe loads in cwts. distributed, we have the simple formula $W' = \frac{bd^2}{L}$. Example: Required, the safe distributed load in cwts. on a fir beam, 14 ins. deep, 9 ins. wide, and 14 ft. span, $W' = \frac{bd^2}{L} = \frac{9 \times 14^2}{14} = 126$ cwts.

CHAPTER II

Transverse Strength of Timber—Safe Loads on Floors—Designing Flitched Beams

The formulæ given in the previous chapter, and any similar formula, can be transposed so that, any one item being unknown, it can be readily found. For example, it is required to know what depth a fir beam should be to carry a distributed load of $1\frac{1}{2}$ cwts. per foot run when it is 3 ins. wide and the span is 12 ft. Then

$$W' = \frac{bd^2}{L},$$

or, by transposition,

$$d^2 = \frac{W'L}{b},$$

this being effected by multiplying both sides of the equation by L and dividing by b, cancelling where the same letters occur above and below the division line. Putting figures for the letters and turning the load per foot run into total load by multiplying it by the 12 ft. span, we have

$$d^2 = \frac{1\frac{1}{2} \times 12 \times 12}{3} = \frac{216}{3} = 72,$$

and the square root of 72 is approximately $8\frac{1}{2}$, which is the required depth in inches.

More difficult questions may be worked with a little consideration. For example, it is required to know over what span a 9-in. by $1\frac{1}{2}$-in. scaffold board will safely carry a man weighing 168 lbs. Now, a live or moving load, applied suddenly, may be assumed to cause double the stress that an equal dead load would do, so that W must be taken as if it were

$$\frac{168 \times 2}{112} = 3 \text{ cwts.}$$

By transposition,

$$L = \frac{cbd^2}{W},$$

but taking $c = 3{\cdot}5$, as before, and factor of safety 4, will make the formula

$$L = \frac{3{\cdot}5\ bd^2}{4\ W} = \frac{3{\cdot}5 \times 9 \times 1{\cdot}5^2}{4 \times 3} = 5{\cdot}9, \text{ say } 6 \text{ ft.}$$

The factor of safety will depend upon circumstances; it is an allowance chiefly for unforeseen contingencies, and timber is subject to knots, shakes, and decay; so that a factor of safety of 4 may be looked upon as the least that can be allowed in any case. A table for general use may be drawn up as follows:—

Factors of Safety for Timber.

Dead loads	For temporary use	4
	Permanent use under cover	7
	Permanent use exposed to weather	10
Live loads	Stress in one direction only	10
	Equal alternating stresses	15
	Variable with shocks	20

The constant c used for timber will vary with the kind of timber employed; it is approximately the load in cwts. in centre required to break a unit beam 1 in. square placed on supports 1 ft. apart, but it may vary 50 per cent. in any given case owing to the variable nature of the material. The following is a summary showing the constant adopted for timbers in general use.

Constants for Timber.

Being c in the formula $W = \frac{cbd^2}{L}$;

8 for greenheart;
6 for ash;
5·5 for American elm;
5 for English oak, pitch pine, teak, mahogany;
4·5 for beech, jarrah, kauri pine;
4 for Baltic oak, Dantzic, Memel;
3·5 for spruce fir, larch, Riga;
3 for English elm.

There is a very simple rule in use by architects for the scantlings of floor joists for dwelling houses and ordinary buildings,

viz., half the span in feet plus 2 gives the depth in inches, and the thickness is about one-third of the depth. For example, with a clear span of 14 ft., we have $\frac{14}{2} + 2 = 9$ ins. as the depth, and $\frac{9}{3} = 3$ ins. the proper thickness. Speculative builders will, of course, use smaller scantlings than this, but nothing less should be used in good work. Warehouses will require special consideration. The load provided for is made to vary with the class of building, and the following may be taken as average amounts.

SAFE LOADS ON FLOORS.

Dwelling houses, upper floors . .	$\frac{3}{4}$ cwt. per ft. super.
Dwelling houses, principal floors .	1 ,, ,,
Churches and public buildings . .	$1\frac{1}{2}$,, ,,
Warehouses and factories . .	$2\frac{1}{2}$ to 5 ,, ,,

Where the floor construction is of a heavy character, as with pugging in wood floors, and all fire-resisting floors, the weight of the floor must be allowed in addition to the foregoing.

In designing a floor the thickness of the boards will theoretically depend upon the distance apart of the joists, but practically the joists are almost invariably 12 ins. apart, and the boards are made of a thickness just sufficient to withstand accidents and to suit the pocket of the builder: for warehouses a surplus thickness is allowed for wear. Taking 1 cwt. per ft. super as the load, and 12 ins. apart for the joists, the calculated thickness necessary for the boards would be

$$d = \sqrt{\frac{W'L}{b}} = \sqrt{\frac{1 \times 1}{12}} = \cdot 288,$$

or only a trifle over $\frac{1}{4}$ in.; but a concentrated load of 1 cwt. might come upon a single board 7 ins. wide at the centre between two joists. So that a more correct calculation for the floor boards would be

$$d = \sqrt{\frac{2\,WL}{b}} = \sqrt{\frac{2 \times 1 \times 1}{7}} = \cdot 534,$$

or a trifle over $\frac{1}{2}$ in.; while generally they are not made less than $\frac{5}{8}$ in. for upper floors and $\frac{3}{4}$ in. for principal floors, but in better

class work they may be found $\frac{3}{4}$ in. and 1 in. respectively. The proper size of joists for any given case may be calculated from the formula

$$bd^2 = \frac{sW'L^2}{12},$$

W' being the load in cwts. per foot super, s the spacing of joists in inches from centre to centre, and the other letters as before. For example, a floor is to carry $1\frac{1}{4}$ cwts. per foot super, the joists to be 16 ft. span and 15 ins. from centre to centre. Then

$$bd^2 = \frac{sW'L^2}{12} = \frac{15 \times 1\frac{1}{4} \times 16^2}{12} = 400.$$

Suppose the depth to be fixed at 9 ins., the necessary thickness will be $\frac{400}{9^2} = 5$ ins., or, if the depth be 11 ins., the thickness will be $\frac{400}{11^2} = 3\frac{1}{3}$. In timber work it is always necessary to consider the market sizes; therefore, in the present case, it would probably be taken as 9 ins. by $4\frac{1}{2}$ ins. or 11 ins. by 3 ins. The latter would form a stiffer floor, although the strength would be about the same; but the former would save 2 ins. in the thickness of the floor and in the height of the walls. The strength of timber varies as the square of the depth, but the stiffness varies as the cube of the depth, so that when stiffness is required it is desirable to have the depth as great as possible.

The allowable deflection of floors in order to avoid the cracking of ceilings is $\frac{1}{4}$ in. per 10 ft. of span, or $\frac{1}{480}$ of the span. The deflection of timber is given by the formula $D = \frac{WL^3}{cbd^3}$, where D = deflection in centre in inches, W = load in cwts. in centre, ends supported, c = constant = teak 50, Quebec oak 40, fir and deal 33, Dantzic oak 27, pitch pine 25. The proportionate deflection for a given span, according to the conditions of loading and supporting, will be as follows:—

Comparative Deflection under same Total Load.

Supported both ends, load central	= 1
,, ,, load distributed	= $\frac{5}{8}$
Fixed one end, loaded the other	= 16
,, load distributed	= 6

When timber is required to be used in a large size, as for a bressummer, it is desirable to cut it down the centre, reverse one piece end for end, and bolt the two pieces together with the cut surfaces outside. This equalises the strength, promotes the seasoning, and by putting the heart wood outside reduces the chances of decay. The pieces when so cut are called "flitches," and the bolted beam would be a flitched beam. Fig. 11 shows a half elevation of the complete beam and Fig. 12 the section. Theoretically the strength is not altered, and the formulæ already given may be used in calculating the safe load; but practically the risks of failure are reduced, so that if great economy were required, the factor of safety adopted could be smaller. Beams of this character are generally made suitable for carrying heavier loads by the insertion of a steel flitch plate.

In order to exemplify the designing of flitched beams it will be well to consider the case of, say, a flitched beam of Memel fir with a steel plate, to carry 8 tons distributed over a clear span of 14 ft. The weight of the finished beam should be allowed for, and this will be about

$$\frac{WL}{120} = \frac{8 \times 14}{120} = \cdot 93, \text{ say 1 ton.}$$

A useful approximate formula for a flitched beam of fir and steel with a distributed load is

$$W' = \frac{(b + 7t)\, d^2}{L},$$

where W' = safe total load in cwts. distributed, b and d = breadth and depth of timber in inches, t = thickness of flitch plate in inches, L = span in feet. In this case

$$W' = 9 \times 20 = 180 \text{ cwts.} \quad \text{and} \quad L = 14 \text{ ft.};$$

therefore $$d^2\,(b + 7t) = 180 \times 14 = 2{,}520.$$

Try $d = 12$, $b = 12$ for total breadth of timber only, then the necessary thickness of flitch plate will be

$$12 \times 12\,(12 + 7t) = 2{,}520 \quad \text{or} \quad 1{,}728 + 1{,}008t = 2{,}520,$$

or $$1{,}008t = 2{,}520 - 1{,}728,$$

whence $$t = \tfrac{792}{1008} = 0{\cdot}8,$$

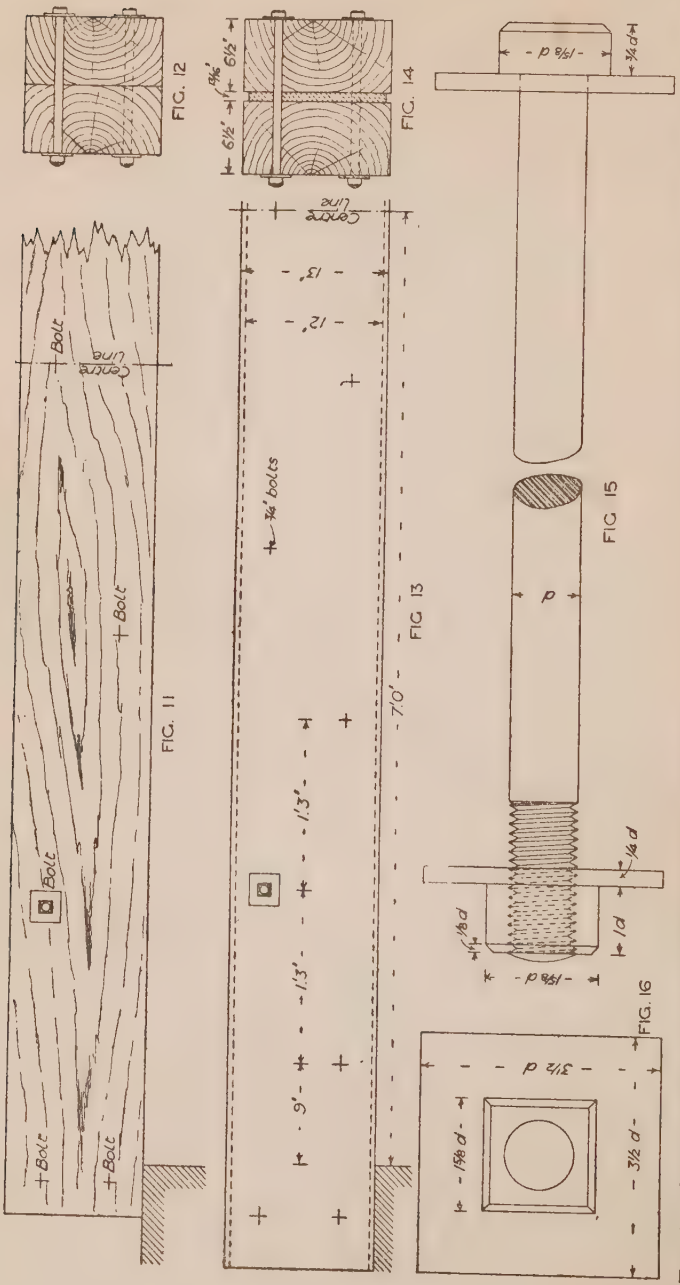


FIG. 11.—Half Elevation of Simple Flitched Beam. FIG. 12.—Section of same. FIG. 13.—Half Elevation of Flitched Beam, with Steel Flitch Plate. FIG. 14.—Section of same. FIG. 15.—Elevation of Standard Bolt. FIG. 16.—End view, showing Nut and Washer.

for which a $\frac{13}{16}$ in. plate = ·8125 will be suitable. To prevent the beam from rocking on the edge of the plate by the shrinkage of the timber, it will be necessary to increase the depth of the timber to, say, 13 in., which is the size Memel usually runs to, and using a whole timber the width would also be 13 in.

The design can be checked by another formula, viz.,

$$W = \frac{d^2}{L}(cb + 42t),$$

where W = breaking load in cwts. in centre, b and d = breadth and depth of timber in inches, c = constant, say 3·5 for fir, t = thickness of steel flitch plate in inches. Taking the dimensions deduced from the previous formula

$$W = \frac{13^2}{14}(3{\cdot}5 \times 13 + 42 \times {\cdot}8125);$$

but to be correct, as the flitch plate is not so deep as the beam, in multiplying out we must make the allowance for the difference, thus

$$W = \frac{13^2}{14} \times 3{\cdot}5 \times 13 + \frac{12^2}{14} \times 42 \times {\cdot}8125 = 549 + 351 = 900 \text{ cwts.}$$

breaking load in centre. Divide by 10 for factor of safety = 90 cwts. safe load in centre, and multiply by 2 for distributed load = 180 cwts. = 9 tons safe load distributed, which is exactly what we have to provide for.

In most books the instructions terminate here, but in practical designing there are a few other points to consider. The working pressure upon the bearing area for fir must not in any circumstances exceed 250 lbs. per sq. in. The load at each end will be

$$\frac{9 \times 2{,}240}{2} = 10{,}080 \text{ lbs.},$$

then $\frac{10080}{250} = 40{\cdot}3$ sq. ins. bearing area required, but the width is 13 ins., therefore the length of beam on the bearing surface must be at least $\frac{40{\cdot}3}{13} = 3{\cdot}1$ ins., and in practice $4\frac{1}{2}$ ins. would be the minimum allowed—not so much to reduce the pressure on the timber as to carry the load a little further in from the edge of the bearing surface; in fact, it would be an advantage

to give a length of bearing of 9 ins. There is no general rule for the size or spacing of the bolts; they are left to individual judgment. They should not be nearer than 3 ins. to the edge of the timber nor less than $\frac{5}{8}$ in. diameter, and usually $\frac{3}{4}$ in., two in a vertical line at each end and others placed zigzag about 18 ins. to 2 ft. apart throughout the length, avoiding, if possible, the placing of a bolt on the lower side in the centre, as that would weaken the beam. All holes in steel plates should be drilled, as punching stars the hole and damages the plate, and holes cannot be punched unless they are about $\frac{1}{8}$ in. larger in diameter than the thickness of the plate. Fig. 13 shows a half elevation of the completed beam, and Fig. 14 the section.

It is important that the bolts in a structure should be properly proportioned and fixed. Too often one sees the nuts so thin that there is every chance of stripping the thread when tightening up; and many carpenters think that a washer is only required under the nut to prevent tearing the fibres of the wood in screwing up, whereas washers are required under both head and nut to spread the pressure over a greater area and reduce the risk of crushing the fibres and working loose. The following table gives the standard proportions for bolts and nuts for all purposes where timber is concerned:—

PROPORTIONS OF BOLTS IN CARPENTRY.

Threads per inch (Whitworth): $\frac{1}{2}$ in. = 12, $\frac{5}{8}$ in. = 11, $\frac{3}{4}$ in. = 10, $\frac{7}{8}$ in. = 9, 1 in. = 8, $1\frac{1}{4}$ in. = 7, $1\frac{1}{2}$ in. = 6.

Thickness of nut	= 1	diameter of bolt	
,, ,, head	= $\frac{3}{4}$	,,	,,
Diameter of head or nut over sides	= $1\frac{5}{8}$	,,	,,
Side of square washer for fir	= $3\frac{1}{2}$	,,	,,
,, ,, ,, hard wood or when let in flush in fir	= $2\frac{1}{2}$	,,	,,
Thickness of washer	= $\frac{1}{4}$	,,	,,

The nearest $\frac{1}{16}$ in. may be taken in any of the measurements. The length of a bolt is usually taken as the distance from the underside of the head to the point. The "wood measure" or "grip" of a bolt is the distance between the washers when the

nut is fully on the bolt. The screw thread should continue at least one diameter further down the shank than would appear to be required in order to allow for the shrinkage of the timber. Fig. 15 shows the elevation and Fig. 16 end view of a standard bolt with nut and washers. The dimensions are given in fractions of the diameter, so that they apply to all cases. For example, if a $\frac{3}{4}$ in. bolt be wanted, thickness of nut = 1 diam. = $\frac{3}{4}$, thickness of head = $\frac{3}{4}$ diam. $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$ in., width over sides $1\frac{5}{8}$ diam. = $1\frac{5}{8} \times \frac{3}{4} = \frac{13}{8} \times \frac{39}{32} = 1\frac{7}{32}$ in. or 1·21875 or practically $1\frac{1}{4}$ ins. and so on.

CHAPTER III

Distribution of Stress in Beams—Bending Moments and Shear Stresses—Transverse Strength of Timber

It will now be necessary to go a little more deeply into the question of the distribution of stress in beams. The simplest case that can be taken is a cantilever loaded at the end, as Fig. 17. It is easy to see that this acts like a lever, and that the stress upon the beam increases in proportion to the distance from the load, but the process involves two distinct sets of things, covered by Newton's third law of motion, "action and reaction are equal." There is, first, the action of the load in producing various intensities of stress at different parts of the beam, and then the reaction of the fibres in resisting the stress depending upon their area and position. At present we will deal with the first of these, as regards the longitudinal stresses of tension and compression and the transverse stress of shear. The loads on a beam produce what are called bending moments. A bending moment is like all other moments, in being the product of a force into a leverage, and in the case of the beam whatever cross-section be considered, the forces, whether loads or reactions, tend to turn the end of the beam clockwise or anti-clockwise round the section. The bending moment at the section is the algebraic sum, that is, the arithmetical difference of the clockwise and anti-clockwise moments. The longitudinal stresses are not bending moments, but are measured by them. It is only necessary to divide the bending moment at any point by the effective depth of the beam to obtain the total stress of tension or compression at that point, and that again divided by the effective area above or below the neutral axis gives the intensity of the stress.

In a cantilever loaded at the end the bending moment at any point is the distance from the load to that point, multiplied by

the load. If x be the distance of the point from the support the bending moment will be $W(L - x)$, and the bending moment will be a maximum at the support $= WL$. Fig. 18 shows a graphic diagram of the bending moments on a cantilever loaded at the end. This being drawn to scale and placed directly under the cantilever the bending moment at any point can be easily measured on the vertical ordinates. The thick line shows the theoretical beam, as the result is quite independent of the material of the beam or its construction, whether solid, rectangular, or circular, or hollow, or flanged. Fig. 19, in the same way, shows the shear diagram. Shear diagrams are, as a rule, very easily set off. At the support the shear stress is equal to the reaction, or the amount of load borne by that support, and in a cantilever the whole load comes on the one support, therefore the shear stress is equal to the load. Then passing along the beam, whatever the nature of the loading, the shear stress is reduced by the amount of load passed. In the cantilever the load passing through the beam remains the same up to the end, where, on passing the load, the shear is reduced to nil, because the value at support minus the load reduces the shear to nothing.

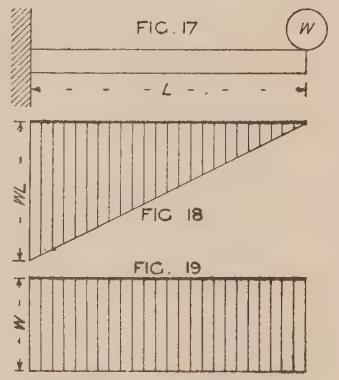


FIG. 17.—Cantilever loaded at end.
FIG. 18.—Bending moment diagram.
FIG. 19.—Shear diagram.

The same principles are followed in the case of a cantilever with a uniformly distributed load, as Fig. 20. If the load be divided up into portions, the bending moment produced by each will be calculated exactly as in the case of the concentrated load; but this would be a roundabout method, and as all the results would have to be added together the diagram would appear as Fig. 23. If the load were divided into sufficiently small portions the outline of the bending moments would be seen to be a semi-parabola, so that we may construct it on this principle, as shown in Fig. 21. The simplest way to construct

the parabola is shown in Fig. 24, where the length of the rectangle is equal to the length of cantilever, and the height equal to the maximum bending moment. Then any number of equal divisions are taken along the bottom, and the same number up the side, vertical lines being drawn from the base, inclined lines are drawn from the divisions at the side to the vertex at the opposite end, to intersect the vertical lines and give points in

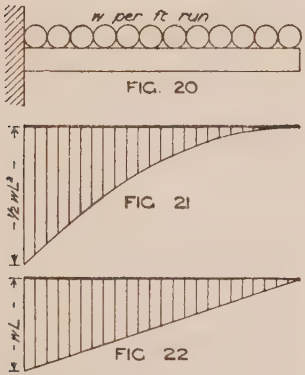


FIG. 20.—Cantilever with Distributed Load.
FIG. 21.—Bending moment diagram for Fig. 20.
FIG. 22.—Shear diagram for same.

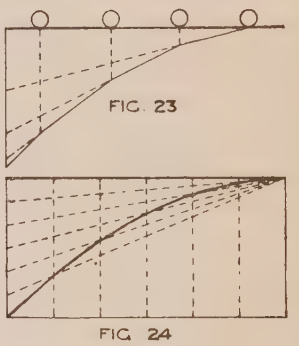


FIG. 23.—Distributed Load on Cantilever taken in separate portions.
FIG. 24.—Construction of Parabola.

the parabola, which points being joined by a curve the whole outline is obtained. A distributed load is generally given as so much per foot run $= w$, and as the centre of gravity of a distributed load on a cantilever is only half the distance from the support the maximum bending moment will be $\frac{1}{2}(wL)\,L = \frac{1}{2}wL^2$. The shear diagram follows the rule already laid down, and will, therefore, commence at the support $= wL$, reducing by w as each foot is passed, and giving a triangular outline. With several detached concentrated loads the principle shown in Fig. 23 would be followed.

In the case of a beam supported at the ends and loaded in the centre, as Fig. 25, the bending moment diagram will be as Fig. 26, with a maximum of $\frac{WL}{4}$ immediately under the load. The

shear diagram will be as Fig. 27, commencing with $+\frac{1}{2}W$ on the edge of left-hand support, then remaining constant until the load is reached, when it reduces by the amount of load and becomes $-\frac{1}{2}W$. The + and — are merely useful conventions, and there is no real difference in the shear stress, which is equal on each side of the centre. With a uniformly distributed load on a beam supported at the ends, as Fig. 28, the bending

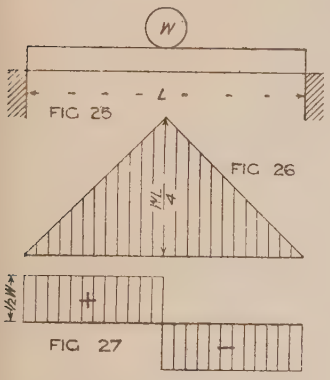


FIG. 25.—Beam supported at ends and loaded in centre.
FIG. 26.—Bending moment diagram for Fig. 25.
FIG. 27.—Shear diagram for same.

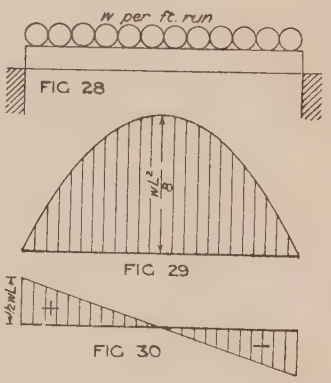


FIG. 28.—Beam supported at ends with Distributed Load.
FIG. 29.—Bending moment diagram for Fig. 28.
FIG. 30.—Shear diagram for same.

moment diagram will be parabolic, as Fig. 29, the maximum being $\frac{wL^2}{8} = \frac{WL}{8}$, and therefore only half of that produced by an equal concentrated load. The shear diagram, constructed as already explained, will be as Fig. 30, showing that there is no stress in the centre, while the maximum is at each support $= \frac{1}{2}wL = \frac{1}{2}W$, the same as was found with an equal concentrated load in the centre. With a uniformly distributed load, and also a concentrated load, as Fig. 31, the two bending moment diagrams may be combined, as Fig. 32. The concentrated load being out of the centre, dividing the length into x and y, the maximum bending moment due to that load will be $W\left(\frac{x \times y}{x + y}\right)$. The shear diagram, Fig. 33, looks rather peculiar, but is constructed as before;

the maximum occurring at left-hand support will be $\frac{1}{2}\ wL$, due to the distributed load, $+\ W\left(\frac{y}{y+x}\right)$, due to the concentrated load, being the reaction at the support. There are two remarkable properties in the relationship of the bending moments and shear stresses which should be observed and tested. They are, first, that the bending moment is at a maximum value where the shear is at a minimum, and *vice versâ*; and second, that the bending moment at any point is equal to the *area* of the shear diagram from left-hand abutment up to that point, bearing in mind the plus and minus values. Figs. 29 and 30 are easy examples to test in this way.

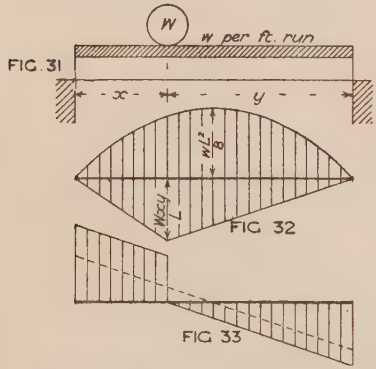


FIG. 31

FIG. 32

FIG. 33

FIG. 31.—Beam supported at ends with Distributed Load and Concentrated Load out of centre.

FIG. 32.—Bending moment diagram for Fig. 31.

FIG. 33.—Shear diagram for same.

Now we must turn to the other side of the question, and deal with the resistance set up in the beam. For this we must know the shape of the beam, and may take first a uniform solid rectangular section, as Fig. 35. Draw two diagonals by joining the opposite corners, and put shading lines across the middle triangles. Then, remembering that the longitudinal stress is greatest at the top and bottom surfaces, and reduces to nothing at the neutral axis, the shaded portion may be looked upon as giving by its horizontal width at each part the comparative value of the stress, or it may be looked upon as giving the amount of material in use if it were all equally stressed; with this in view it is sometimes called the inertia area, *i.e.*, the area of effective resistance. The "moment of inertia" is the summation of the areas of all the individual fibres or parts in the cross-section of a beam multiplied by the squares of their distances from the neutral axis, $I = \Sigma\ ay^2$. Taking Fig. 35, the moment of inertia of the section is $D\ A\ G + d\ a\ g$ where D and d are the depth above and below neutral axis,

A and a the area of each shaded portion, G and g the distance of centre of gravity of each from neutral axis. It will be seen that $D\ A\ G + d\ a\ g = 2\ D\ A\ G$, and using b and d for breadth and depth of beam the moment of inertia may be given in terms of

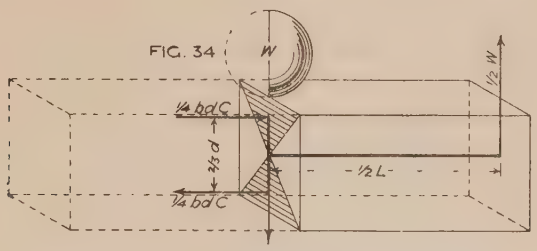


FIG. 34.—Graphic diagram of bending moment and moment of resistance in a Beam.

the dimensions, then, $2\ DAG = 2 \times \frac{1}{2}\ d \times \frac{1}{4}\ bd \times \frac{2}{3}\ (\frac{1}{2}\ d) = \frac{bd^3}{12}$, which is the moment of inertia for any rectangular section. If the moment of inertia of a section be divided by the depth

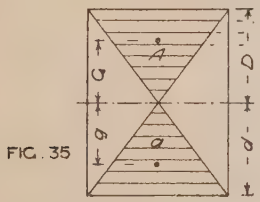


FIG. 35. — Distribution of Longitudinal Stress in Rectangular Beam.

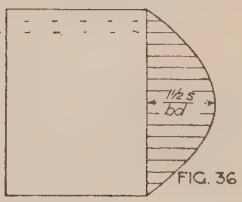


FIG. 36.—Distribution of Shear in Rectangular Beam.

from the neutral axis to the top or bottom, which is generally known as y, it will give the "section modulus" Z, or $\frac{I}{y} = Z$. But $y = \frac{1}{2}\ d$, therefore $Z = \frac{bd^3}{12 \times \frac{1}{2}\ d} = \frac{1}{6}\ bd^2$. The section modulus may be described as the resistance-value of any section depending upon the area and the disposition of its parts.

Now as regards "the extreme fibre stress." Theoretically this is equivalent to the maximum stress in tension or compression, but it does not coincide either with the tensile strength or with the compressive strength, and its value varies with the form of the cross-section of the material. Several theories have been propounded in explanation of this peculiarity, though none of them is entirely satisfactory. Rankine assumes one cause to be the fact that the resistance of a material to direct stress is increased by preventing or diminishing the alteration of its transverse dimensions. He also suggests that when a bar of metal is torn asunder the strength indicated is that of the centre part, which is the weakest, whilst when it is broken transversely the strength indicated is that of the outer part, which is the strongest. In the case of timber it is suggested that the lateral adhesion of the fibres prevents the outer ones from moving freely, and hence in all cases the actual extreme stress is considerably less than it appears by calculation. In any event, the difference really exists, and instead of determining the modulus of rupture from the tensile and compressive strength, it can only be found by actual experiment on cross-breaking. *The modulus of rupture for transverse strength* is C in the formula $\frac{WL}{4} = ZC$ (W being in lbs. and L in inches); while c in the formula $W = \frac{cbd^2}{L}$ (W being in cwts., b and d in inches, and L in feet) may be called *the coefficient of transverse strength.* The values of c have already been given; those for C are given in the following table, and are applicable to a beam of any section, the effect of the form of section being provided for by Z:—

Moduli of Rupture for Transverse Strength (C).

Ash	12,000	to	18,000 lbs.
Beech	9,000	,,	12,000 ,,
Chestnut	—	,,	10,600 ,,
Deal, Christiania	—	,,	9,370 ,,
Elm, Canadian	—	,,	14,490 ,,
,, English	4,700	,,	— ,,
Fir, Baltic	4,900	,,	10,000 ,,

Fir, Spruce	9,000	to	12,300 lbs.
Greenheart	16,500	,,	27,500 ,,
Larch	5,000	,,	10,000 ,,
Mahogany, Honduras	11,500	,,	12,600 ,,
Maple	—	,,	10,160 ,,
Oak, Quebec	10,000	,,	12,600 ,,
,, English	9,600	,,	13,600 ,,
Pine, red	7,000	,,	10,300 ,,
,, white	7,400	,,	— ,,
,, yellow	7,100	,,	8,450 ,,
,, Dantzic	8,550	,,	13,800 ,,
,, Memel	8,000	,,	— ,,
,, pitch	—	,,	14,090 ,,
,, Riga	8,300	,,	9,450 ,,
Teak	12,000	,,	19,000 ,,

In general, $C = 18c \times 112$; C is the so-called extreme fibre stress $= K$ of Molesworth, k of Tredgold, and f of other writers, and c, as before stated, is the weight in cwts. in centre required to fracture a bar 1 in. square and 1 ft. long.

A useful comparison of the various values may be made in the case of cast iron, as follows:—

Average tensile strength per sq. in.	= 16,500 lbs.
,, compressive strength per sq. in.	= 99,000 ,,
,, resistance to shearing per sq. in.	= 27,700 ,,
,, modulus of rupture (C) per sq. in.	= 44,000 ,,
Coefficient of transverse strength (c)	= 2,445 ,,

It should be noted in the case of cast iron that C varies from the average tensile strength in flanged beams to $2\frac{1}{3}$ or $2\frac{1}{2}$ times that amount in rectangular beams. The moment of resistance of the section is dependent upon the area of the section, the disposition of its parts, and the strength of the material measured by the so-called "extreme fibre stress." Then the relationship of the bending moment to the moment of resistance is shown by the following equations, where any manner of loading may be taken; but, for example, we will take a concentrated load in the centre.

Bending moment = Moment of resistance.

$$\frac{WL}{4} = \frac{I}{y}C$$

or $$\frac{WL}{4} = ZC$$

or $$\frac{WL}{4} = \frac{1}{6}bd^2C$$

This can be transposed in various ways as $W = \frac{\frac{2}{3}Cbd^2}{L}$, or $C = \frac{1\frac{1}{2}WL}{bd^2}$. If we adopt the principle of leverage the relationship of bending moment and moment of resistance can perhaps be shown more clearly. The two sets of forces, as shown in Fig. 34 form two "couples," a couple consisting always of two equal parallel forces, and the value of a couple in mechanics is one of the forces multiplied by the distance between the pair. The reaction at the support, and the half load transmitted through the beam to that support, form the bending moment couple and the resistance to tension and compression acting at the centre of gravity of each form the moment of resistance couple, thus

$$\tfrac{1}{2}W \times \tfrac{1}{2}L = \tfrac{1}{4}bd \times C \times \tfrac{2}{3}d$$

whence $$\frac{WL}{4} = \tfrac{1}{6}bd^2C$$

as previously shown. The shaded portion of the section, which we have previously called the inertia area, is sometimes known as "the modulus figure," as it is the graphic representation of the section modulus Z.

The shear diagrams already shown give the whole shear across any section, but the shear stress is not distributed equally throughout the section. Fig. 36 shows the distribution throughout a rectangular section. If the horizontal lines in Fig. 35 be looked upon as dividing up the shaded portion into individual parts, then the shear ordinate in Fig. 36, opposite the bottom of the first part, will be equal to the area of that part; the next shear ordinate will be equal to the combined area of the first and second part, and so on to the neutral axis. Below the neutral axis the area of each part will be deducted, so that the shear stress is zero at top and bottom of the section, giving a parabola

for outline. It can be proved mathematically from this diagram that the maximum shear at the centre of the section is $1\frac{1}{2}$ times the mean shear, that is, the total shear across the section being s, the mean shear will be $\frac{s}{bd}$ and the maximum shear per sq. in. will be $\frac{1\frac{1}{2}\ s}{bd}$. It will thus be seen that the longitudinal stresses are at a maximum at the top and bottom surfaces where the shear is zero, and that the longitudinal stresses gradually reduce to zero at the neutral axis, while the shear increases to its maximum at that layer, and that the whole of the fibres are thus doing approximately uniform work.

CHAPTER IV

Investigation of Stresses in Lean-to Roof—Loads on Roofs—Tensile and Compressive Strength of Timber

It is an unfortunate fact that the simplest roofs structurally are the most difficult forms to deal with when the stresses have to be determined. It is not until we reach the framed trusses that we find the work of calculating the stresses at all easy. The difficulty can be approached gradually if a commencement be made with a simple rafter as a lean-to roof (Fig. 37). With tile or slate battens fixed at the usual distance apart, the load may be taken as equivalent to a uniformly distributed load attached to the rafter. There will be less tendency to slip downwards and cause thrust on the wall if the bearing surface at each end is cut horizontal as shown, but there will still be a thrust. Each individual portion w of the total load W' will act as shown in Fig. 38, but the whole load W' will be the sum of the portions w: therefore, in Fig. 38 W' may be substituted for w, with the result that the load acting perpendicularly to the rafter will be $W' \cos 30° = W' \times \frac{\sqrt{3}}{2}$, and the thrust along the beam at the foot will be $W' \sin 30° = W' \times \frac{1}{2}$. Now, if we had a horizontal beam of span S with the same load W', we should have the maximum bending moment $\frac{W'S}{8}$. Calling L the length of the rafter,

$$L = S \sec 30° = S \times \frac{2}{\sqrt{3}},$$

but the load acting perpendicularly to the length of rafter is $W' \times \frac{\sqrt{3}}{2}$, so that the maximum bending moment on the inclined rafter will be

$$\frac{W' \times \frac{\sqrt{3}}{2} \times S \times \frac{2}{\sqrt{3}}}{8} = \frac{W'S}{8},$$

as with a beam of the same horizontal span.

There is still the thrust to take account of before we can determine the necessary dimensions of the rafter. The thrust will be nil at the top and increase in proportion of w sin 30° for each interval passed, until the bottom is reached, when it will be $= W''$ sin 30° $= W'' \times \frac{1}{2}$. Assume the horizontal span to be

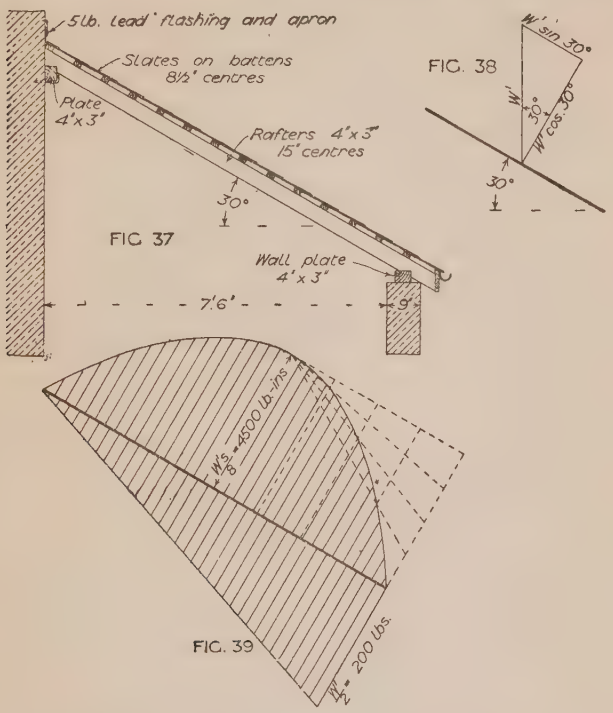


FIG. 37.—Section through Lean-to Roof.
FIG. 38.—Vertical Load producing Thrust and Cross Strain.
FIG. 39.—Bending Moment and Direct Thrust diagram for Fig. 37.

7 ft. 6 ins., and the whole load on one rafter to be 400 lbs., then the maximum bending moment will be

$$\frac{W'S}{8} = \frac{400 \times 7{\cdot}5 \times 12}{8} = 4{,}500 \text{ lb.-ins.}$$

at the centre of the rafter, the remainder of the rafter being under varying bending moments given by the ordinates to the parabola in Fig. 39. The thrust will be given by ordinates to the

triangle in Fig. 39, the maximum being at the bottom $= \frac{W'}{2} =$ 200 lbs. It is useful to show these against the same thick line representing the rafter, but they cannot be measured together as they are in different units.

The combination of a bending moment and a direct thrust is a simple mathematical operation, but it will be worth while to make a slight digression to explain the matter graphically. Fig. 40 shows by the shading the equivalent area for uniform compression and tension, marked + and —, due to the bending moment. Fig. 41 shows to the same scale of intensity the equivalent compression area due to the direct thrust. Then

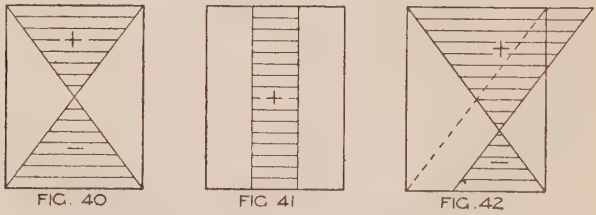


Fig. 40.—Area under Stress by Bending Moment.
Fig. 41.—Area under Stress by Direct Thrust.
Fig. 42.—Combination of Figs. 40 and 41.

Fig. 42 shows the algebraic sum of the two previous diagrams, the compression of Fig. 40 being increased by that of Fig. 41 and the tension reduced by the same amount. By the mathematical method we have the direct thrust portion represented by $\frac{W}{A}$, and the bending moment portion by $\frac{M}{Z}$, the complete formula being $\frac{W}{A} \pm \frac{M}{Z}$, the + value, or sum, giving the maximum compression in Fig. 42, and the — value, or difference, giving the maximum tension. The maximum stresses will occur in the centre of the rafter, and, assuming the section to be 4 ins. by 3 ins., $W = 100$ lbs., $M = 4{,}500$ lb.-ins., then $\frac{W}{A} \pm \frac{M}{Z} = \frac{100}{12} \pm \frac{4500}{\frac{1}{6}(3 \times 4^2)} = 8\frac{1}{3} \pm 562\frac{1}{2} = 570{\cdot}83$ lbs. per sq. in. compression, and 554·16 lbs. per sq. in. tension.

As the stresses in such a rafter under the same load per foot run, and at the same pitch, will vary approximately with the square of the span, we may take the calculations just made as a standard, and, say, bd^2 varies as S^2, or 3×4^2 varies as $7{\cdot}5^2$, which gives a ratio of $\frac{3 \times 4^2}{7{\cdot}5^2} = \frac{48}{56{\cdot}25} =$ say $\frac{6}{7}$. Then, for a span of 10 ft., $bd^2 = 10^2 \times \frac{6}{7} = 85{\cdot}7$, which would be given approximately by $3\frac{1}{2}$ ins. by 5 ins.; and for any other span, in the same

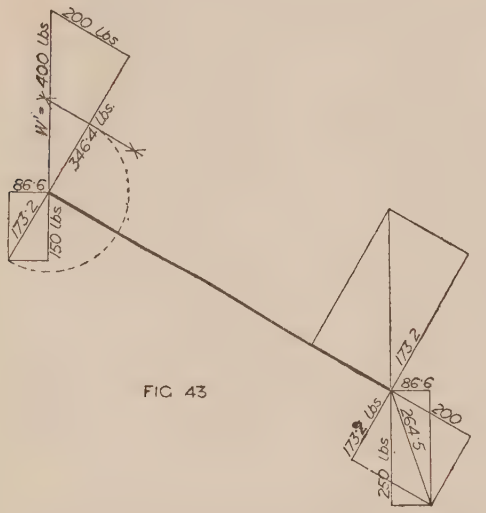


FIG. 43

FIG. 43.—Forces and Reactions at Ends of Rafter.

way, six-sevenths of the square of the span in feet will give the product of the breadth by the square of the depth for the required rafters.

So far, the effect of the load on the rafter only has been considered, but it is necessary to follow the matter a little further and ascertain how the supports are affected. This will be best worked graphically as in Fig. 43. Set up the whole load vertically at the upper end of the rafter to any given scale; construct the triangle of forces by adding lines parallel and perpendicular to the rafter. Of the whole load perpendicular to the rafter half will act at the top and half at the bottom. Produce

the line through the top support, and cut it off equal to the half-load 173·2 lbs.; then complete the parallelogram, and it will be found that the horizontal thrust on the support will be 86·6 lbs., and the vertical load on the support will be 150 lbs. At the bottom end of rafter there will be two active forces, viz., the 200 lbs. thrust and 173·2 lbs. perpendicular force similar to the top. Produce these through the point of support and combine them into a parallelogram of which the diagonal or resultant will scale 264·5 lbs., which is the combined result of thrust and load on the lower support. This may now be resolved into horizontal and vertical directions as shown, giving 86·6 lbs. horizontal thrust at bottom, the same as at top, and 250 lbs. vertical load, which, with the 150 lbs. supported at the top, makes the whole load of 400 lbs. carried on the rafter. The load of 400 lbs., which has been taken, is more than the actual weight of covering in order to allow for the effect of a high wind without complicating the calculations.

In dealing with a pent- or span- roof consisting of a series of pairs of rafters without any tie or collar beam, the conditions will be very similar to the last, with the addition of a thrust at the ridge down each rafter from the other one. Before proceeding with an example of this, it will be well to consider how the load upon a roof is made up, measuring the sloping surface for area.

Dead Load on Roofs.

	Per sq. ft.
Slating, ordinary	8 lbs.
Slate battens	2 „
Slate boarding	3 „
Tiling, plain, including laths	18 „
„ pan „ „	12 „
„ Italian (ridge and furrow)	14 „
Corrugated iron, 20 W.G.	2 „
Lead, including laps (7-lb. lead)	$7\frac{1}{2}$ „
Zinc, 14 gauge	$1\frac{1}{2}$ „
Lath and plaster ceiling	8 „
Ceiling joists	3 „
Common rafters	3 „

	Per sq. ft.
Snow, to be allowed when the wind is taken less than 28 lbs.	5 lbs.
Wind, average force perpendicular to roof plane when sheltered	21 ,,
,, ,, partially sheltered	28 ,,
,, ,, exposed	42 ,,
,, ,, much exposed	56 ,,
Purlins and trusses (wood)	10 ,,
,, ,, ,, (iron)	7 ,,

Or more accurately for iron and steel = ·75 (span ft.)^2.

If we double the arrangement shown in Fig. 37, making the junction as shown in Fig. 44, we shall have an ordinary span roof with a clear span of 15 ft. The dead load may be taken from the table as follows: Slating, 8 lbs.; battens, 2 lbs.; rafters, 3 lbs.; total, 13 lbs.; wind on one side, 28 lbs. The length of each rafter will be

$$7{\cdot}5 \text{ sec. } 30^\circ = 7{\cdot}5 \times \frac{2}{\sqrt{3}} = 7{\cdot}5 \times 1{\cdot}155 = 8{\cdot}66 \text{ ft.},$$

and the distance from centre to centre may be taken as 15 ins. = 1·25 ft.; then the load on each will be 8·66 × 1·25 × 13 = 140 lbs. structural load, and 8·66 × 1·25 × 28 = 303 lbs. wind on one side. Take first the effect of the structural load. It is clear that, apart from any added effect due to the pressure of one rafter against the other, there will be stresses similar to Fig. 39, substituting the loads of 140 lbs. instead of the 400 lbs. there taken, making a bending moment in the centre of each rafter of

$$\frac{140 \times 7{\cdot}5 \times 12}{8} = 1{,}575 \text{ lb.-ins. and thrust of } \frac{140}{2} = 70 \text{ lbs.}$$

Looking at Fig. 43, it will be seen that the load normal to each rafter is at the apex,

$$\frac{W' \cos 30^\circ}{2} = \frac{140 \times \frac{\sqrt{3}}{2}}{2} = 60{\cdot}62 \text{ lbs.}$$

The wind may be taken on, say, the right hand side, half at the top and half at the bottom = $\frac{303}{2}$ = 151·5 lbs. Then, adding this to the amount already on the right-hand side at the top, we

have 60·62 + 151·5 = 212·12 lbs., while on the left there is only the 60·62 lbs. Combining these into a parallelogram, as shown in Fig. 45, we have the resultant, 248·05 lbs. Transferring

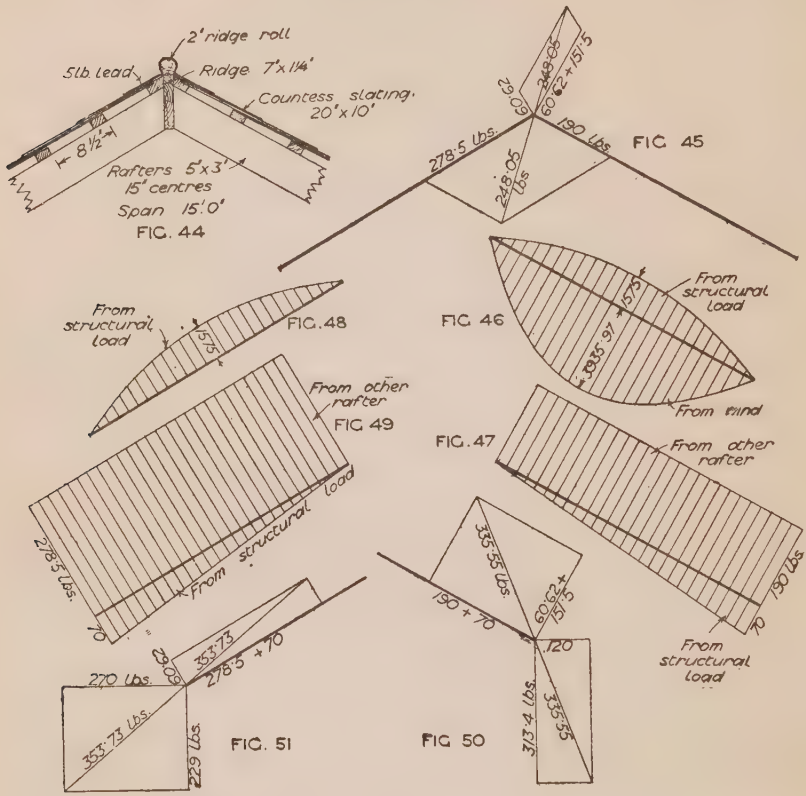


FIG. 44.—Junction of Rafters at Ridge of Pent Roof. FIG. 45.—Forces and Reactions at Ridge. FIG. 46.—Bending Moments on Right-hand Rafter. FIG. 47.—Thrust diagram for Right-hand Rafter. FIG. 48.—Bending Moment diagram for Left-hand Rafter. FIG. 49.—Thrust diagram for Left-hand Rafter. FIG. 50.—Forces and Reactions at Foot of Right-hand Rafter. FIG. 51.—Forces and Reactions at Foot of Left-hand Rafter.

this to the inside of the roof, and completing the new parallelogram by drawing lines from the extremity parallel to each of the rafters, and scaling off, we have a thrust of 190 lbs. down the right-hand rafter, and 278·5 lbs. down the left-hand rafter,

produced by the wind, in addition to the thrust produced by the load.

Besides these forces, we have the bending moment on the right-hand rafter produced by the wind. The length of the rafter is, as shown previously, 8·66 ft., and the load 303 lbs.; the bending moment in the centre will therefore be

$$\frac{W'L}{8} = \frac{303 \times 8{\cdot}66 \times 12}{8} = 3{,}935{\cdot}97 \text{ lb.-ins.}$$

These results may now be combined as regards the stress on the rafters. For the right-hand rafter, Fig. 46, we have the combined bending moment of 1,575 + 3,935·97 = 5,510·97, say, 5,511 lb.-ins., and the combined thrust shown in Fig. 47 is 225 lbs. in the centre. Then for the maximum stress on the rafter, assuming a 5-in. by 3-in. section,

$$\frac{W}{A} \pm \frac{M}{Z} = \frac{225}{15} \pm \frac{5511}{\frac{1}{6} \times 3 \times 5^2} = 15 \pm 440{\cdot}9 = 455{\cdot}9 \text{ lbs.}$$

per sq. in. compression and 425·9 lbs. per sq. in. tension. For the left-hand rafter we have the bending moment diagram Fig. 48, with a maximum of 1,575 lb.-ins., and for the thrust, diagram Fig. 49, giving 313·5 lbs. in the centre. Then the maximum stress will be

$$\frac{W}{A} \pm \frac{M}{Z} = \frac{313{\cdot}5}{15} \pm \frac{1575}{12{\cdot}5} = 20{\cdot}9 \pm 126 = 146{\cdot}9 \text{ lbs.}$$

per sq. in. compression, and 105·1 lbs. per sq. in. tension. As the wind may blow from either side, the results upon the rafter with the greater stress will decide whether the section chosen will be sufficient.

The ultimate tensile and compressive strengths of timber are given in the table on page 34, and the lower values should be used unless special care is taken in the selection, when the average values may be adopted. The factor of safety will be taken as already given, according to the circumstances under which it is used.

Now we have to consider the result upon the walls. Taking the right-hand rafter, the thrust of 190 lbs. from the left-hand

ULTIMATE TENSILE AND COMPRESSIVE STRENGTH OF TIMBER IN LBS. PER SQ. IN.

	Tension.	Compression.
Ash	12,000 to 17,000	8,600 to 9,300
Beech	11,000 „ 22,000	7,700 „ 9,300
Chestnut	9,000 „ 13,000	—
Deal, Christiania	12,000	5,850
Elm, Canadian	9,200	9,200
„ English	4,480	5,600
Fir, Baltic	3,360 to 12,000	2,500 to 5,500
„ Spruce	2,900 „ 12,000	5,000 „ 6,800
Greenheart	8,000 „ 9,200	12,000 „ 15,200
Larch	4,000 „ 11,000	3,200 „ 5,500
Mahogany, Honduras	3,000 „ 18,000	6,000 „ 8,000
Maple	10,600 „ 15,400	6,000
Oak, Quebec	6,720 „ 12,000	6,000 to 6,900
„ English	6,000 „ 19,000	6,400 „ 10,000
Pine, red	2,700 „ 14,000	4,700 „ 7,500
„ white	3,360 „ 11,800	4,100 „ 5,000
„ yellow	2,000 „ 12,000	4,000 „ 8,000
„ Dantzic	3,100 „ 10,000	5,400 „ 6,900
„ Memel	9,400 „ 11,000	6,500 „ 13,440
„ pitch	4,700 „ 12,000	4,700 „ 5,300
Teak	3,300 „ 15,000	8,500 „ 12,000

[Although these figures are compiled from the best authorities, they show very considerable variation, but possibly not more variation than may be found in actual practice.]

rafter shown at the ridge in Fig. 45 is transmitted down the rafter and combined at the foot with the thrust of $\frac{W'}{2} = 70$ lbs., as shown in Fig. 50. Then, also acting at the foot, will be the half of the wind pressure 151·5 lbs. + half the normal component of structural load 60·62 lbs. = 212·12 lbs. Combining these into a parallelogram, the resultant will be found to be 335·55 lbs., and producing this through the point of support and

setting off the equilibrant an equal length beyond, a new parallelogram can be constructed by resolving the diagonal into horizontal and vertical reactions as shown. The left-hand rafter will be treated in an exactly similar manner, as shown in Fig. 51. As the wind may come from either side, the walls must be strong enough to resist either pair of reactions.

CHAPTER V

Elementary Principles of Trigonometry—Reciprocal Diagrams of Stress—Designing a Trussed Beam Footbridge

As in these articles occasional reference will be made to the trigonometrical functions of sine, cosine, etc., it will probably be of service to give a diagram showing their relationship. Fig. 51A shows a circle with the trigonometrical functions or

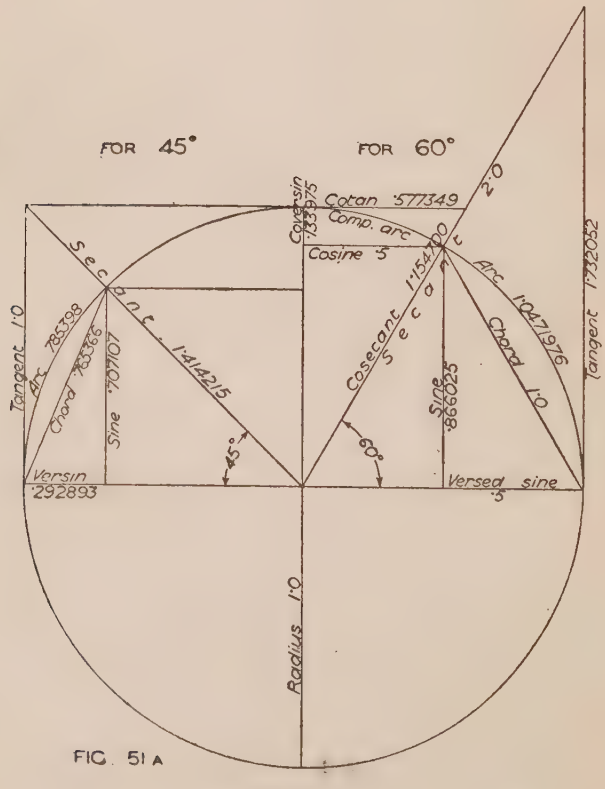


FIG. 51A

Fig. 51A.—Diagram of Trigonometrical Functions.

ratios and their values for angles of 45 and 60 degrees. They are called ratios because they result from the proportion of two parts of the triangle as follows—

Trigonometrical Ratios.

$$\frac{\text{Perpendicular}}{\text{Hypothenuse}} = \text{Sine} \qquad \frac{\text{Base}}{\text{Hypothenuse}} = \text{Cosine}$$

$$\frac{\text{Perpendicular}}{\text{Base}} = \text{Tangent} \qquad \frac{\text{Base}}{\text{Perpendicular}} = \text{Cotangent}$$

$$\frac{\text{Hypothenuse}}{\text{Base}} = \text{Secant} \qquad \frac{\text{Hypothenuse}}{\text{Perpendicular}} = \text{Cosecant}$$

Where one of the components of these ratios is unity, the line marked on the circle may be called by the name of the ratio and simply measured off.

This is not the place to enlarge further upon the subject of trigonometry, but full information can be obtained from "Practical Trigonometry" (Whittaker & Co., 2*s*. 6*d*. net).

In order not to meet all the difficulties together a slight digression may now be made, going back to timber beams. A simple footbridge of 15 ft. span and 5 ft. clear width may be required, with a light handrail on each side. As it may be fully loaded on occasions, it is necessary to know the weight of a crowd of men. This has been frequently tested by experiment, and very different results are given by the various experimenters.

Weight of a Crowd of Men in lbs. per foot super.

G. G. Page . .	35¼	Parsey . .	80—112
French practice .	41	Nash . .	120
American practice .	45	W. K. Kernot	126
Hatfield . . .	70	W. C. Kernot	143·1
F. Young . .	80	B. B. Stoney .	147·4
T. Page . . .	84	L. J. Johnson	181·3

The latter results have been obtained by employing specially-selected men, packed in a way in which no accidental crowd could ever exist, and it will be sufficient to consider 1 cwt. per foot super as ample to allow for footbridges and floors where

the only load is a human one. Although it is a "live" load no extra allowance need be made, as no movement affecting the stresses could be made when the persons are so closely packed as they would be. The floor of the proposed bridge being 15 ft. $\times$ 5 ft., the total external load will be $15 \times 5 \times 1 = 75$ cwts. Add to this an assumed load of 12 cwts. for the structure, then the load to be carried by each beam will be $\frac{75 + 12}{2} = 43{\cdot}5$ cwts. Taking the formula for safe load distributed as $W' = \frac{bd^2}{L}$, we have $bd^2 = W'L = 43{\cdot}5 \times 15 = 652{\cdot}5$, and, assuming the breadth to be 9 ins., the depth will be $= \sqrt{\frac{652{\cdot}5}{9}} = \sqrt{72{\cdot}5}$ $= 8{\cdot}514$, or say 9 ins., making the beams 9 ins. by 9 ins. The flooring, placed transversely across the top of the beams, would generally be made 3 ins. thick without calculation, but it can easily be seen if this is sufficient. The clear span being 3 ft. 6 ins., $W' = \frac{bd^2}{L} = \frac{12 \times 3^2}{3{\cdot}5} = 30{\cdot}8$ cwts. distributed and $\frac{30{\cdot}8}{3{\cdot}5} = 8{\cdot}8$ cwts. per foot super safe load, allowing considerable margin for wear. The whole design would then be as Fig. 52 half elevation, and Fig. 53 cross-section.

Now let the span be doubled, and the load will, in consequence, be doubled also; then, as we have seen previously in the case of the comparative strength of rafters for various spans, the stress to be resisted will vary as the square of the span, or four times the stress found for the bridge calculated above. Doubtless timbers could be obtained of sufficient scantling for this purpose, but it will be instructive to adopt a new design by which timbers of moderate scantling can still be used. Let the type be a single trussed beam, as Fig. 54, with, say, 12-in. by 9-in. beams, with cast-iron strut and wrought-iron tie rods. The beam is here shown in skeleton outline only, and is in the form known as a "frame diagram." The load is uniformly distributed over the beam, but for the purpose of ascertaining the stresses by the method described below it is necessary to divide the load up over the points at which the beam is

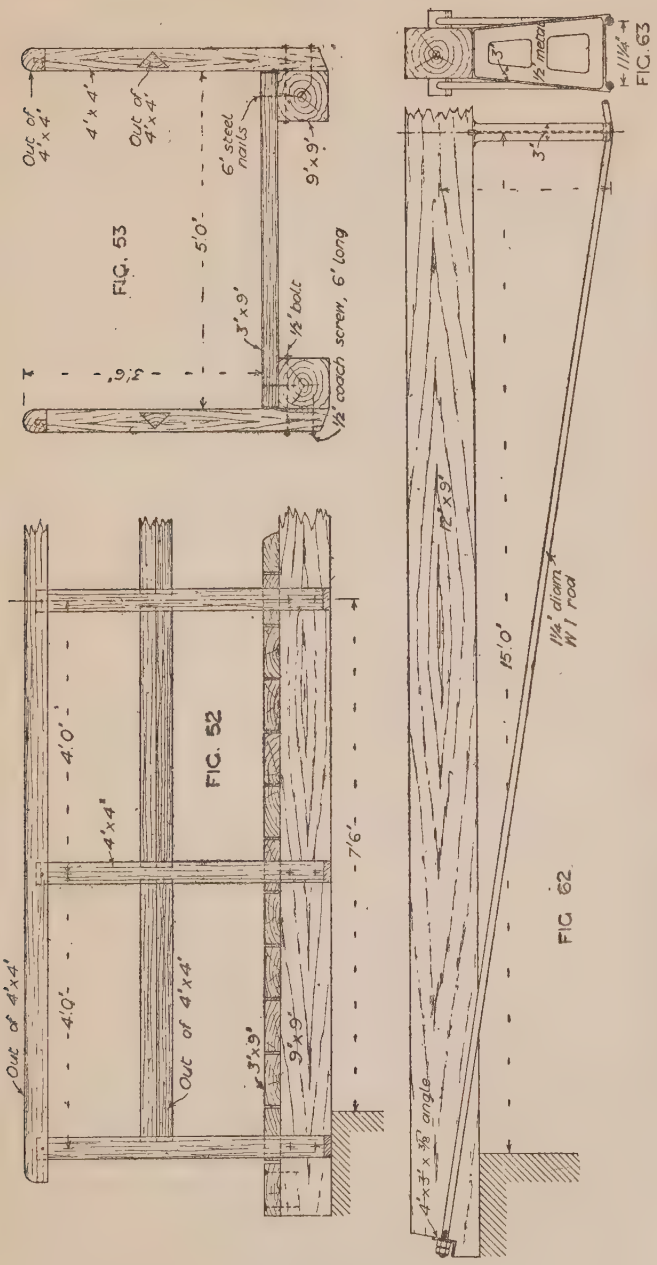


FIG. 52.—Half Elevation of Simple Footbridge. FIG. 53.—Cross-Section of Simple Footbridge.

FIG. 62.—Half Elevation of Trussed Beam for Footbridge. FIG. 63.—Cross-Section of Trussed Beam for Footbridge.

supported. Taking the load at double that in the previous case, it will be 87 cwts. on each beam, and as half the load will be upon each half of the beam by the ordinary reckoning, it will give one-fourth at each end $= \frac{87}{4} = 21{\cdot}75$ cwts., and two-fourths in the centre $= 43{\cdot}5$ cwts., and the reactions at each end will be equal to half the total load $= 43{\cdot}5$ cwts., as shown.

The stresses in the various parts of the beam could be found by using the parallelogram of forces, beginning at one abutment

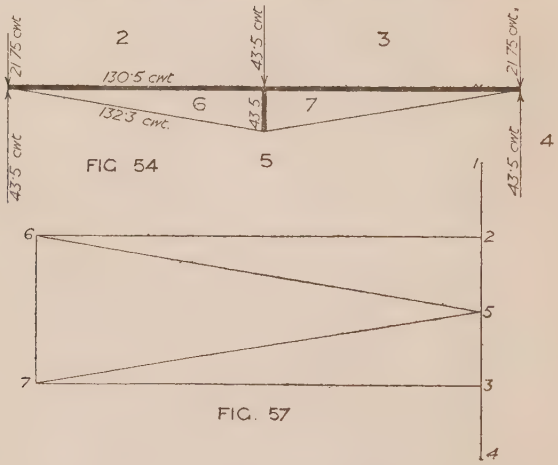


Fig. 54.—Frame diagram for Trussed Beam.
Fig. 57.—Stress diagram for Fig. 54.

and finishing at the opposite one. There would not be much difficulty in doing this in the present case, but it will be instructive to apply the method known as that of "reciprocal diagrams," which is used exclusively when more complicated cases have to be worked. The principle of reciprocal diagrams can be shown most easily by considering three forces in equilibrium upon a point, as Fig. 55, where the forces are represented in magnitude by the length of the lines, in direction or line of action by their position, and in "sense," *i.e.*, to or from the point, by the arrow heads. Now, if these three forces be taken in order and be made to run concurrently, they will form a triangle of forces (Fig. 56), and the fact of the triangle

closing will show that the forces are in equilibrium. The method of numbering as indicated is known as "Bow's Notation." It consists in numbering consecutively each of the spaces between the forces in Fig. 55, each force being then known by the number of the space on each side; thus, the first force will be known as 1-2, the second 2-3, and the third 3-4. This is called the frame diagram. Then the reciprocal diagram (Fig. 56) will be constructed by drawing force 1-2 from point 1 to point 2, parallel to its position in Fig. 55. Then from 2 to 3 will be drawn force 2-3, and from 3 to 4 force 3-4. The term "reciprocal diagrams" is used because the forces that meet in a point in one of them form a closed figure in the other, and *vice versâ*. These principles hold good in every case. Applying the system to Fig. 54, the spaces will be numbered as shown, commencing at the left-hand side and working round clockwise, until the spaces between all the external forces are numbered. The same procedure takes place with the internal spaces, as there will be forces of stress acting in each of the members. To construct the reciprocal diagram or stress diagram (Fig. 57) commence with the external forces, or line of loads and reactions, or it may be called simply "load line" 1 to 2, 2 to 3, 3 to 4, 4 to 5, and 5 to 1, the loads being 1-2-3-4, and the reactions 4-5-1, overlapping because they are all parallel forces. The next point in order on Fig. 54 is 6, and, working always from the known to the unknown, and remembering that the similarly numbered lines in the stress diagram must be parallel to those in the frame diagram, draw 2-6 and 5-6 in Fig. 57, intersecting at point 6, which is thus fixed. Then 3-7 and 5-7 intersecting in

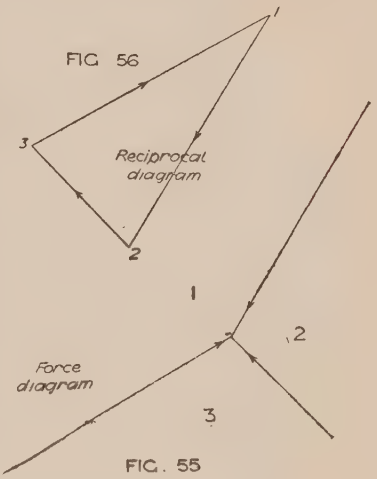


FIG. 55.—Three Forces in Equilibrium.
FIG. 56. — Corresponding Triangle of Forces.

7, and 6-7 is then joined and found to be parallel with 6-7 in the frame diagram. As Fig. 56 only closed because the lines showed the true magnitude of the forces, the lines in Fig. 57 will show the magnitude of the forces in the correspondingly-numbered lines of the frame diagram, and by simple scaling the amount of each stress may be determined. The stresses scaled off from Fig. 57 may now be inserted on Fig. 54 as shown. The next

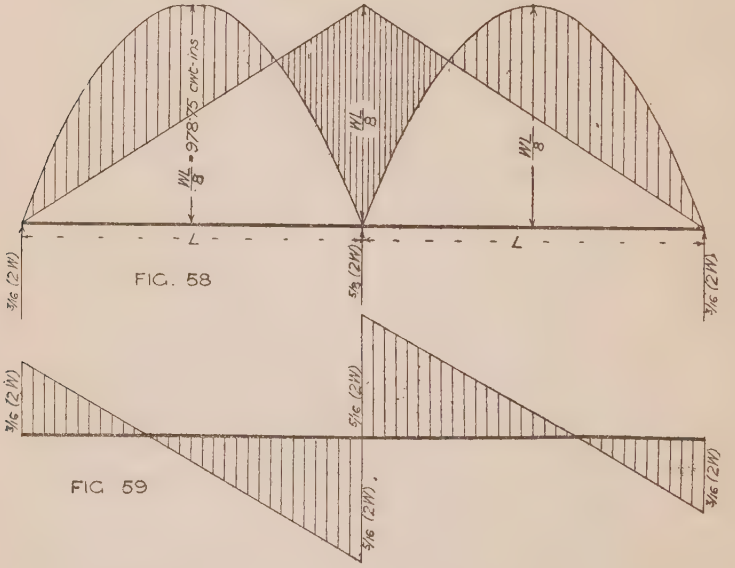


FIG. 58

FIG. 59

FIG. 58.—Bending Moment diagram for Beam continuous over two Spans.
FIG. 59.—Shear diagram for Beam continuous over two Spans.

step will be to determine whether the 12-in. by 9-in. beam will be sufficient, and the first thing we observe is that, besides the direct stress of 130·5 cwts., there is a bending moment produced in each half between the centre and the end due to the load being distributed over it. This may be taken as

$$\frac{W'L}{8} = \frac{43{\cdot}5 \times (15 \times 12)}{8} = 978{\cdot}75 \text{ cwt.-ins.}$$

The length must be in inch units, because it is usual to take the section modulus in inch units. Then the direct stress and

the bending moment are combined into one formula with the section modulus, as explained in a previous chapter, giving $\frac{W}{A} \pm \frac{M}{Z} = \frac{130{\cdot}5}{12 \times 9} \pm \frac{978{\cdot}75}{\frac{1}{6}(9 \times 12^2)} = 1{\cdot}2 \pm 4{\cdot}53 = +\ 5{\cdot}73$ cwts. per sq. in. maximum compression and $-\ 3{\cdot}33$ cwts. per sq. in. maximum tension.

The above is the ordinary method of working out the stresses, but it is not quite correct. The beam being continuous over the central support, is in the condition of a "continuous beam," and

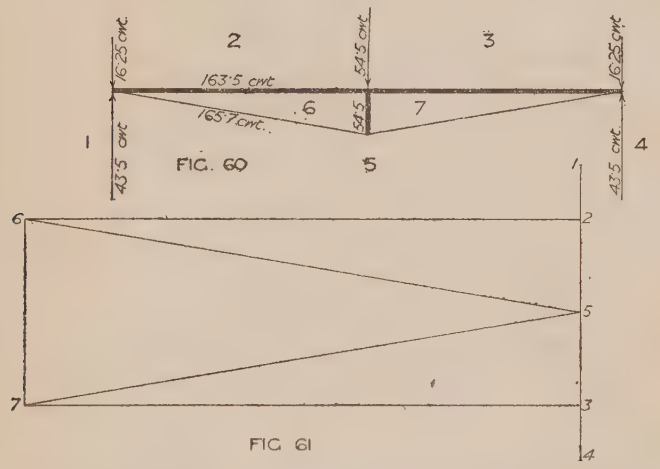


Fig. 60.—Frame diagram of Trussed Beam with Altered Loading.
Fig. 61.—Stress diagram for Trussed Beam with Altered Loading.

the distribution of the loading is not strictly what appears in Fig. 54. The typical bending moment diagram for a beam continuous over two spans is shown in Fig. 58, and the corresponding shear diagram is shown in Fig. 59. The calculations for these diagrams are marked upon them, W being the distributed load on one span and L the length of one span. The closer shading in the centre shows the reversed stresses over the middle support due to bending, viz., tension in upper fibres and compression in lower, the shading lines in each case giving the ordinates for the measurement of the stresses.

From this it will appear that the proper division of the load

over the three supports is $\frac{3}{16}W$, $\frac{5}{8}W$, $\frac{3}{16}W$, as may be proved by the "theorem of three moments," and a new diagram with this division of the load is shown in Fig. 60. The reciprocal diagram corresponding with this is given in Fig. 61, and the stresses scaled off are marked on Fig. 60. From this we can now proceed to design the complete bridge. The maximum stresses in the beam will be $\frac{W}{A} \pm \frac{M}{Z} = \frac{163{\cdot}5}{12 \times 9} \pm \frac{978{\cdot}75}{\frac{1}{6}(9 \times 12^2)}$ $= 1{\cdot}51 \pm 4{\cdot}53 = +6{\cdot}04$ cwts. per sq. in. maximum compression and $-3{\cdot}02$ cwts. per sq. in. maximum tension. These amounts are within safe limits; if they had not been it would have been necessary to assume other dimensions and go through the work again.

Take the tie rods next. The tensile stress in them, by scaling from Fig. 61, is 165·7 cwts. $= \frac{165{\cdot}7}{20} = 8{\cdot}28$ tons. Wrought iron will stand a working stress of 5 tons per sq. in. on the net dimensions, and it is clear that a single rod would be of an unwieldy size. It must, therefore, be divided over two rods, each having a sufficient diameter at the bottom of the thread. The equation of the area to the stress will be $d^2 \times {\cdot}7854 \times 5 = \frac{8{\cdot}28}{2}$, whence $d = \sqrt{\frac{4{\cdot}14}{{\cdot}7854 \times 5}} = \sqrt{1{\cdot}0543}$ $= 1{\cdot}02$ ins. diameter at bottom of thread $= 1\frac{1}{4}$ ins. diameter tie rods. The half elevation and section of one beam with the trussing will be as shown in Figs. 62 and 63, omitting the hand-rail, which may be similar to Figs. 52 and 53. The strut should be of cast iron, as shown, and the ends of the tie rods held by double nuts against a 4-in. by 3-in. by $\frac{3}{8}$-in. steel angle. The close observer will find many small points for criticism—for example, the span in the diagrams has been taken as the clear span, while the effective span may be found on the average equal to the distance from centre to centre of bearing surfaces. If the rods are screwed up too tightly the beam will "hog" and rest on the extreme ends, whereas if they are left slack the beam will "sag" and rest on the edges of the supports. The angle of the tie rods in the frame diagrams will also not correspond exactly with the elevation, but the

difference in the stresses will be slight. Footbridges frequently have joists framed into the beams to keep them in their proper positions, and diagonal wind bracing between the joists to keep the beams firm; but with care in placing neither of the bridges will require the additions, the spiking down of the planks giving sufficient stability.

CHAPTER VI

Stresses in Posts and Columns—Gordon's Formula—Continuous Beams—Designing a Timber Gantry

Before much progress can be made in designing it will be necessary to consider the stresses in posts, struts, and other compression members. In the case of the trussed footbridges, although the beams were in compression, they were prevented from bending by the connection of the other parts, and especially the flooring, but a post or independent strut is free to bend if

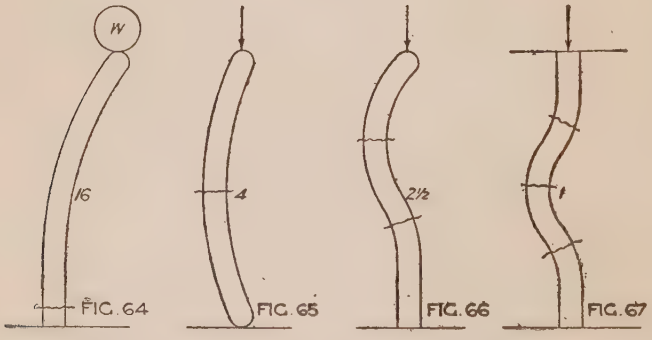


Fig. 64.—Pillar fixed at one end and free at the other.
Fig. 65.—Pillar rounded at both ends.
Fig. 66.—Pillar fixed at one end and rounded at the other.
Fig. 67.—Pillar fixed at both ends.

the stress is sufficiently great. At any rate, there is an incipient tendency to bend which necessitates a reduction of the stress from what is permissible when that tendency is restricted, as in the cases referred to. The tendency to bend varies considerably with the nature of the end fixing, as will be seen by Figs. 64 to 67. Fig. 64 indicates a column, pillar, or strut fixed at the base, and entirely free at the top. This is the condition in which it is most free to bend. Fig. 65 shows a column rounded at both

ends, but with the top load guided in the direction of the arrow. Fig. 66 shows a column with one fixed and one rounded end. Fig. 67 shows a column with both ends fixed. The numbers placed against the middle of each show the comparative liability to bend, and the points of simultaneous fracture are shown by the lines across the figures. Under the term "rounded end" would come compression members with a single bolt or rivet through the end, and the term "hinged" is therefore sometimes substituted for rounded. Under the term "fixed ends" would come not only those that are bolted down by a flange, but simple flat ends when they are of reasonable area, and also those without returned ends if they are fixed by two or more bolts or rivets.

The best-known formula for ascertaining the strength of pillars and compression members is that called Gordon's, because he determined the constants from Hodgkinson's experiments, but the formula was originally due to Tredgold. It is given in almost every formula book, with a varying notation, but leading to the same result. A useful form is

$$p = \frac{f}{1 + \frac{m}{nq}\left(\frac{l}{d}\right)^2}$$

f = ultimate intensity of compressive stress in tons per sq. in. on a short specimen, say for cast iron 36, mild steel 26, wrought iron 18, oak $2\frac{1}{2}$, fir $1\frac{1}{2}$.

p = average thrust or compressive force in tons per square inch on the cross-section of the pillar, which will be the crippling stress when f is taken as above.

l = unsupported length in inches.

d = diameter or least width in inches.

The fraction $\frac{m}{nq}$ is usually given as $\frac{1}{a}$, but then the formula is adapted for only one class of pillar; the present form enables any kind of material or variety of form or fixing to be allowed for.

m = a fixing modulus, say 1 for both ends fixed, $2\frac{1}{2}$ for one end fixed and one rounded, 4 for both ends rounded, 16 for one end fixed and the other free.

n = a shape modulus, say $\frac{1}{4}$ to $\frac{1}{3}$ for built-up sections, $\frac{1}{2}$ for

+ H L or T sections, $\frac{3}{4}$ for solid cylindrical sections, 1 for solid rectangular sections, $1\frac{1}{2}$ for hollow cylindrical sections.

q = a strength modulus, say 500 times the tensile working strength in tons per sq. in.; then:

Material.	Working Strength in Tension per Square Inch.	q =
Fir	$\frac{1}{2}$	250
Oak	1	500
Cast iron	$1\frac{1}{2}$	750
Wrought iron	5	2,500
Mild steel	$7\frac{1}{2}$	3,750

The best factor of safety for metal pillars or struts is that known as Shaler-Smith's $= 4 + \cdot 05\,\frac{l}{d}$. For timber posts the factors already given may be adopted.

An example of the use of the formula may now be taken, say a fir post, under cover, 10 ft. high and 8 ins. by 8 ins. cross-section, flat ends; required the safe load in tons.

$$p = \frac{f}{1 + \frac{m}{nq}\left(\frac{l}{d}\right)^2} = \frac{1{\cdot}5}{1 + \frac{1}{1 \times 250}\left(\frac{10 \times 12}{8}\right)^2} = \frac{1{\cdot}5}{1{\cdot}9}$$

$= \cdot 789$ ton per sq. in. crippling load.

Then the section being 8 ins. by 8 ins., the ultimate load will be $\cdot 789 \times 8 \times 8 = 50{\cdot}496$, say 50 tons. Allowing a factor of safety of 7, the safe load will be $\frac{50}{7} = 7{\cdot}14$, say 7 tons, without any allowance for natural decay beyond the factor of safety. If the angles of the post be chamfered, as is frequently done, the sectional area will be reduced and the carrying capacity lessened, but the balance of advantages and disadvantages will be in favour of the chamfering.

When a post carries a beam there is usually found at the top of the post a stump tenon, as Fig. 68, or a short ordinary tenon, as Fig. 69, or a modification between the two, with a corresponding mortice in the beam. The disadvantage of Fig. 68 is that,

after the post is fixed, it is impossible to see if the tenon is there, or how it fits. Fig. 69 has the advantage that it can be seen from outside, and the beam is not unduly weakened by the mortice if the tenon is a good fit. It is very rarely that a tenon bears at the bottom of the mortice, and the bearing area of the post is therefore reduced by the area of the tenon. This does not seriously affect the strength of the post, as such, and does not enter into the calculations for crippling stress. It chiefly affects the intensity of pressure upon the underside of the beam, which in the case of fir and deal should never exceed, say, 250 lbs. per sq. in., and should be limited to half that amount when possible. As the post has to carry 7 tons =

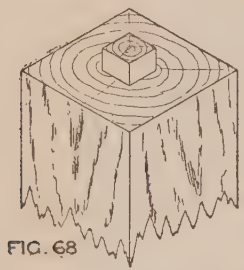


FIG. 68

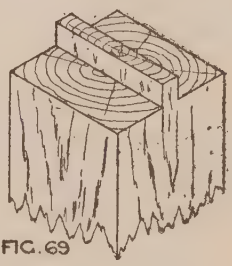


FIG. 69

FIG. 68.—Stump Tenon on Post.
FIG. 69.—Ordinary Tenon on Post.

15,680 lbs., and the bearing area, $8 \times 8 = 64$, minus the tenon, say either 3×3, or $8 \times 1\frac{1}{8} = 55$ sq. ins., the pressure per sq. in. will be $\frac{15{,}680}{55} = 285$ lbs. It is seldom that a post is loaded up to its full capacity, so that possibly no great risk would be run by the slight excess of pressure shown beyond the limit laid down. A corbel is often placed between the post and the beam, as shown in Fig. 71; this is to strengthen the beam by extending its supports and reducing the span, but it will not affect the question of pressure unless it is made of harder wood, such as oak. An iron plate, say 8 ins. wide by 18 ins. long by $\frac{1}{2}$ in. thick, with a mortice for the tenon, might be used as a corbel to spread the pressure over a larger area of the beam, but it would be cheaper to have a larger section for the post, say 10 ins. by 8 ins.

In comparing the results of calculation by Gordon's formula with actual experiments upon various sections, it was found that they did not agree very well, and a modification was introduced by Rankine which made use of the "radius of gyration" of the section instead of the least width. There are several definitions of the radius of gyration, but the simplest way to put it is to say that it equals the square root of the quotient of the moment of inertia divided by the sectional area, or $r = \sqrt{\frac{I}{A}}$; or, as the square of the radius of gyration appears in the formula, it may be stated as $r^2 = \frac{I}{A}$. The whole formula, which is generally known as the Rankine-Gordon formula, is:

$$P = \frac{fA}{1 + \left(\frac{1}{ac}\right)\frac{l^2}{r^2}}$$

where f = about $\frac{2}{3}$ of the ultimate compressive strength of the material in lbs. per sq. in., say, 26,000 for wrought iron, 40,000 for mild steel, 60,000 for cast iron, 6,000 for oak, and 4,000 for fir.

A = area of cross-section in sq. ins.

a = constant = 1 for column with round ends, 2 for column with one end rounded, 4 for column with fixed ends.

c = constant = 9,000 for wrought iron and mild steel, 1,600 for cast iron, 750 for wood.

l = length of column in ins.

r = least radius of gyration of cross-section.

P = ultimate load in lbs. Factors of safety as before.

Applying this formula to the same post, we have first to find the radius of gyration, for which we require to know the moment of inertia, viz., $\frac{1}{12}\ bh^3$. In this connection b is always taken parallel with the neutral axis, and d or h, which are equivalent letters, are taken perpendicular to it. A post has two neutral axes—one across the centre of gravity of the section in each direction; but, where the post is rectangular, bending is only likely to occur in the direction of the least width, so that b will be the greater side and d or h the lesser. The moment of inertia

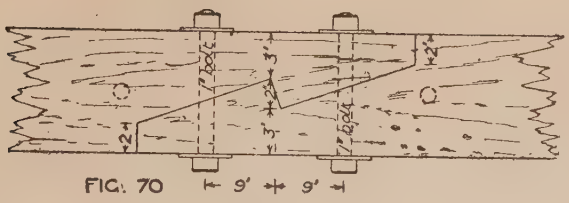


FIG. 70.—Plan of Scarf Joint in Beam.

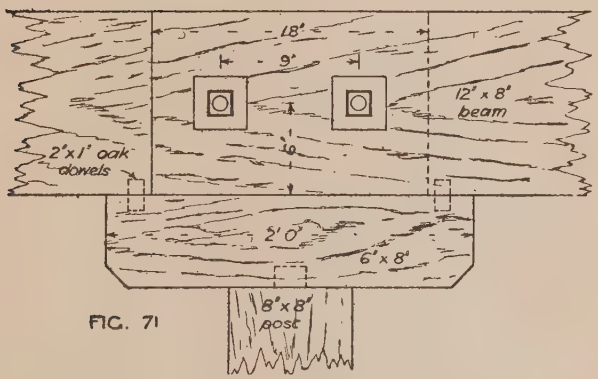
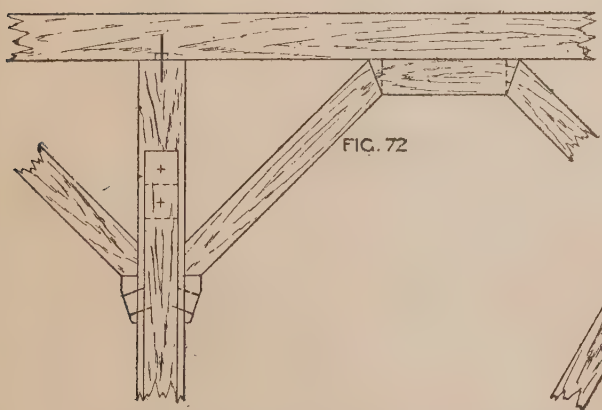


FIG. 71.—Elevation of Scarf Joint in Beam.



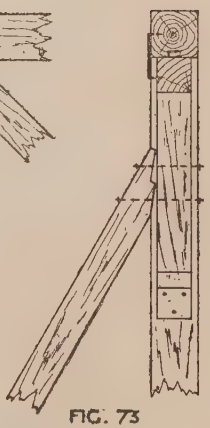


FIG. 72.—Part Elevation of Gantry, showing Struts.
FIG. 73.—Cross Section of Gantry, showing Raking Struts.

being $\frac{1}{12}\,bh^3 = \frac{1}{12} \times 8 \times 8^3 = 341\cdot\dot{3}$, the sectional area $= 8 \times 8 = 64$, then $r^2 = \frac{I}{A} = \frac{341\cdot\dot{3}}{64} = 5\cdot\dot{3}$. Then

$$P = \frac{fA}{1 + \frac{1}{ac}\frac{l^2}{r^2}} = \frac{4{,}000 \times 64}{1 + \frac{1}{4 \times 750} \times \frac{(10 \times 12)^2}{5\cdot\dot{3}}} = \frac{256{,}000}{1 + \cdot 9}$$

$= 134{,}737$ lbs. ultimate load, and a factor of safety of 7 would make the safe load $\frac{134{,}737}{7} = 19{,}248$ lbs. = say, $8\frac{1}{2}$ tons, compared with 7 tons obtained by the simple Gordon formula.

In the above case the safe load on a given post has been ascertained; the converse of that would be when the scantling of a post is to be found to carry a given load. The simplest way will generally be to assume a certain size and calculate its strength, then to enlarge or diminish its section, as may be necessary, and re-calculate.

When beams are continuous over more than one intermediate support they will be calculated as continuous beams, with bending moment diagrams somewhat similar to that for two equal spans, as given in Fig. 58, p. 42. The only difficulty with continuous beams is to know the maximum load transmitted to each support. The following table shows the proportion of the load which each receives when the spans are equal, and the load on each span is unity:—

Load on Supports of Continuous Beams.

Number of Spans.	First Support.	Second Support.	Third Support.	Fourth Support.	Fifth Support.
2	·375	1·250	·375	—	—
3	·400	1·100	1·100	·400	—
4	·393	1·143	·930	1·143	·393

Timber will hardly run more than four spans, and the scarf joints which will come over the posts will be taken as ends simply supported; vertical scarfs are best, as shown in plan (Fig. 70) and elevation (Fig. 71).

For a gantry it will be necessary to strut the posts as shown in Fig. 72, and put raking struts transversely, as shown in Fig. 73.

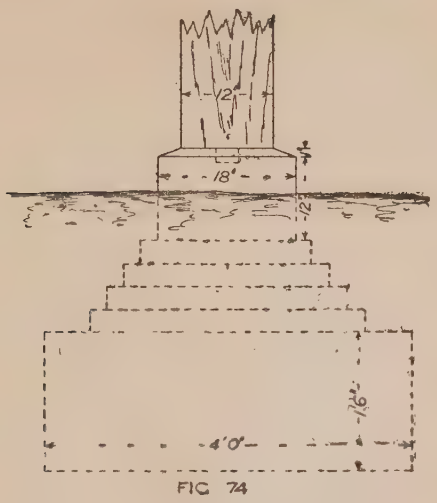


FIG. 74.—Foot of Gantry Post on Stone Base.

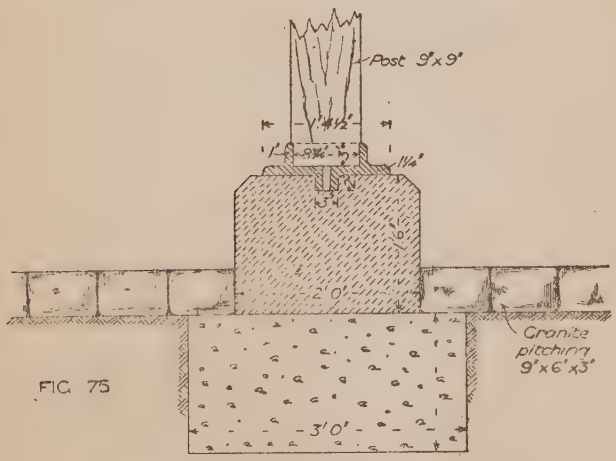
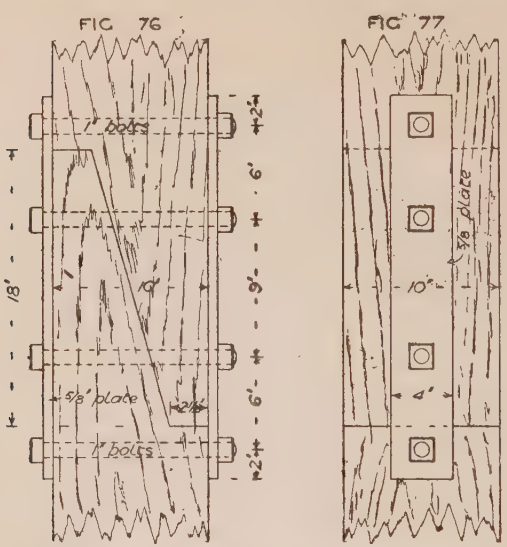


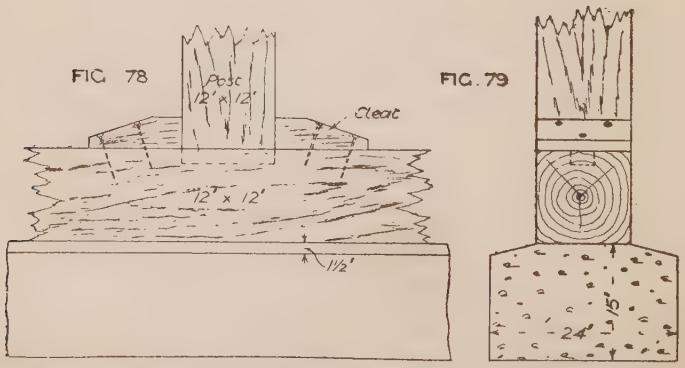
FIG. 75.—Foot of Gantry Post in Cast-Iron Base.

The feet of the posts and raking struts being subject to collisions and rough usage, may be supported as shown in Figs. 74 and 75. A scarf joint in a post should be made as shown in Figs. 76 and



FIGS. 76 and 77.—Front and Side Elevations of Scarf Joint in Post.

77, the bolts and plates rendering the joint practically as strong as any other part. In some gantries a timber sill is laid on concrete to receive the feet of the posts, and cleats should then be put on each side, as shown in Figs. 78 and 79. A rolling load upon a gantry does not produce the same stress as an equally distributed



FIGS. 78 and 79.—Elevation and Section of Foot of Gantry Post on Wood Sill.

load, but it is a little too complex to deal with at present. It may be sufficient to state now that, approximately, each post may be calculated for the full load passing over it, and the beams may be calculated as for the full load resting in the middle of a span, supposing the beam to be merely supported on the adjacent posts and not continuous.

CHAPTER VII

Designing Framed Roof Trusses—Bending Moments on Tie Beam—Details of Joints

Framed roof trusses are particularly well adapted for illustrating the use of reciprocal diagrams. Omitting the unusual case of a "couple close roof," which consists of a pair of rafters united by a tie beam at the feet, and leaving the "collar beam roof" for future investigation owing to the difficulties involved by the use of "virtual forces," a king post truss under vertical loading will be the best form to consider next. Suppose a series of king post trusses to be required for a slated and boarded roof of 25 ft. span. A common distance apart of the trusses is 10 ft., but it will vary more or less according to the length of building to be covered, so that the trusses may be placed at equal intervals. Usually king post trusses would not be less than 7 ft. 6 ins., nor more than 12 ft. 6 ins., apart, and according to the distance apart the load will vary, and also the size of the purlins. The pitch for a slated roof is generally $26\frac{1}{2}$ degrees, which equals a rise of $\frac{1}{4}$ span, or 30 degrees, which facilitates the drawing of the truss and the stress diagrams by the use of a 30-degree set square. The pitch for tiled roofs is generally 45 or 60 degrees, a greater pitch being necessary to throw off the rain more readily, and partly for appearance. In the present case a centre distance of 10 ft. and a pitch of 30 degrees may be assumed. The length of the principal rafters will then be $\frac{1}{2}$ span $\times$ sec $30° = 12{\cdot}5 \times \frac{2}{\sqrt{3}}$ $= 12{\cdot}5 \times 1{\cdot}155 = 14{\cdot}4$ ft. The load per foot super will be, slates 8 lbs., boarding 3 lbs., common rafters 3 lbs., purlins and trusses 10 lbs., wind (average vertical load on straight rafter roofs) 30 lbs., making a total of 54 lbs. The whole upper load on the truss will then be $2 \times 14{\cdot}4 \times 10 \times 54 = 15{,}552$ lbs. = say, 140 cwts. This load must now be divided over the frame diagram,

Fig. 80. Each side will carry $\frac{1}{2}$, each space between the points of support of the principal rafter will carry $\frac{1}{4}$, then as each $\frac{1}{4}$ will

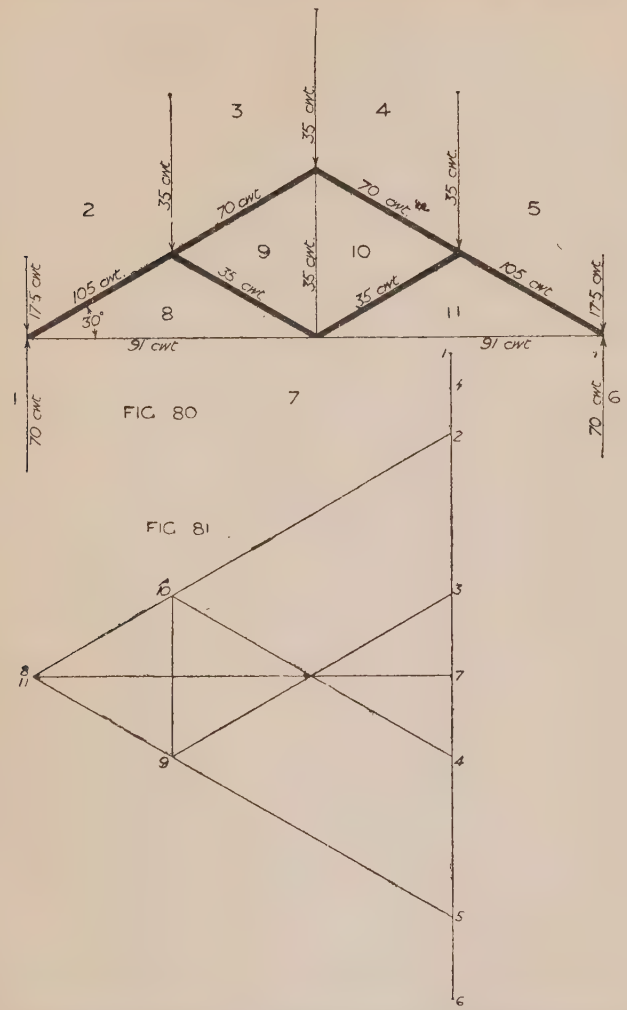


FIG. 80.—Frame diagram of King Post Truss with Vertical Loads only.
FIG. 81.—Stress diagram for Fig. 80.

have two points of support the final division of the load will be $\frac{1}{8}$ over each extremity, and $\frac{1}{4}$ over each intermediate point of support, and the reaction at each wall will be $\frac{1}{2}$, making the

external forces as shown on the frame diagram $\frac{1}{8} \times 140 = 17{\cdot}5$, $\frac{1}{4} \times 140 = 35$, $\frac{1}{2} \times 140 = 70$. The spaces must next be numbered as described in the last chapter, 1 to 11, as shown. The reciprocal stress diagram, Fig. 81, will be constructed as already described. First the load line 1-2-3-4-5-6, then 6-7 and 7-1. Then the force lines parallel to the members of the truss, 2-8, 7-8; 3-9, 8-9; 9-10, 4-10; 10-11, 7-11, and 5-11. It will be seen that one of the points is both 8 and 11, but no difficulty occurs on this account. The length of each of these lines must now be scaled off with the same scale from which the load line was

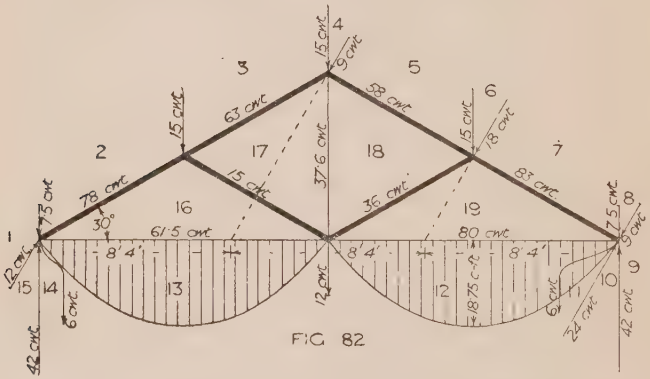


FIG. 82.—Frame diagram of King Post Truss with Structural Load and Wind Pressure.

constructed, and the stresses so found are then marked upon the frame diagram, and the design may be proceeded with. This is the easiest method of ascertaining approximately the stresses in the different members, and errs on the side of safety, but a more accurate way will be to take the wind separately as acting upon one side only, normal to the slope of the roof. To do this a new frame diagram must be made, Fig. 82. The net structural load will now be taken without the wind pressure $= 54 - 30 = 24$ lbs. per ft. sup., and divided up in the same proportion as before, the amount being placed against each of the vertical force lines in the frame diagram, as shown. Then the wind taken normal to the slope of the roof, say 28 lbs. per ft. sup. $= 14{\cdot}4 \times 10 \times 28 =$ 4,032 lbs. $=$ 36 cwts. in all, giving the divisional loads, as 9, 18

and 9 cwts. The reactions may be obtained by constructing a funicular polygon, but it will be better to take this method later on. At present it can be obtained by leverage. Produce each of the force lines representing the wind to the tie beam, as shown by the dotted lines, and scale off the distances, which will be found as marked. By calculation the distances will be $\frac{1}{4}$ span $\times \sec^2 30 = \frac{1}{4} \times 25 \times (\frac{2}{\sqrt{3}})^2 = 6{\cdot}25 \times \frac{4}{3} = 8\frac{1}{3}$, and it will be seen that they divide the tie beam into three equal portions. We require to know what proportion of the wind pressure is borne by each wall; beginning at the right hand side, there is 9 cwts. acting direct: then $18 \times \frac{8\frac{1}{3}}{25} = 6$ cwts. on the left wall, and $18 \times \frac{16\frac{2}{3}}{25} = 12$ cwts. on the right wall; $9 \times \frac{16\frac{2}{3}}{25} = 6$ cwts. on left wall, and $9 \times \frac{8\frac{1}{3}}{25} = 3$ cwts. on right wall, giving reactions from the wind, parallel to the direction taken for it, of $6 + 6 = 12$ cwts. on left wall, and $9 + 12 + 3 = 24$ cwts. on right wall. The addition of a lath and plaster ceiling carried by ceiling joists may be made in this case, the load caused by it being shown as forces acting downwards from the underside of truss. The load per ft. sup. will be: ceiling 8 lbs., ceiling joists 3 lbs., giving a total of 25 $\times$ 10 $\times$ 11 = 2,750 lbs. = say, 24 cwts., divided over the points of support in the proportions of 6 cwts., 12 cwts., and 6 cwts.

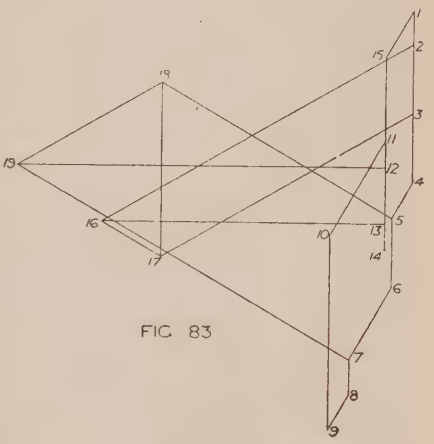


FIG. 83.—Stress diagram for Fig. 82.

The reciprocal diagram, Fig. 83, may now be constructed, taking all the external forces from the frame diagram for the load line, which becomes an irregular polygon, and the other lines being drawn parallel to

the members of the truss complete the stress diagram. There is no difficulty if the work be done systematically, and a detailed description need not be given. Scaling off the values, they may now be put upon the frame diagram, as shown, and the designing will proceed as follows. The usual formula for roof members in compression is the Rankine-Gordon formula, assuming one end to be fixed and the other rounded, except in the case of special connections, where the ends are rigidly held without a possibility of tilting, when they are taken as both fixed. For the principal rafter the maximum stress is seen to be 83 cwts., and the unsupported length is 7·2 ft.

Assume a scantling of 6 ins. by 5 ins., then

$$P = \frac{fA}{1 + \frac{1}{ac}\frac{l^2}{r^2}} = \frac{4{,}000 \times 30}{1 + \frac{1}{2 \times 750}\frac{(7{\cdot}2 \times 12)^2}{2{\cdot}083}} = \text{say, } 36{,}360 \text{ lbs.}$$

In the present case, as all the stresses have been so carefully considered, the factor of safety may be reduced from 7 to, say 4, making the safe load $\frac{36{,}360}{4} = 9{,}090$ lbs., or 81 cwts.

For the struts, with a maximum compression of 36 cwts., and unsupported length of 7·2 ft., try a section 4 ins. by 5 ins., then

$$P = \frac{fA}{1 + \frac{1}{ac}\frac{l^2}{r^2}} = \frac{4{,}000 \times 20}{1 + \frac{1}{2 \times 750}\frac{(7{\cdot}2 \times 12)^2}{1{\cdot}3}} = 16{,}913 \text{ lbs. and}$$

$\frac{16{,}913}{4} = 4{,}228$ lbs., or 38 cwts. safe load.

The king post has a tensional stress of 37·6 cwts., and, allowing 3 cwts. per sq. in. on net area in tension, the king post will require an area of $\frac{37{\cdot}6}{3} = 12{\cdot}5$ sq. ins., say 3 ins. by 5 ins.

The tie beam has a maximum tension of 80 cwts. and maximum bending moment of 18·75 cwt.-ft. = 225 cwt.-ins. Try a 12-in. by 5-in. section, then $-\frac{W}{A} \pm \frac{M}{Z} = -\frac{80}{60} \pm \frac{225}{\frac{1}{6}(5 \times 12^2)} = -1{\cdot}33 \pm 1{\cdot}87 = 3{\cdot}2$ cwts. per sq. in. tension, and 0·54 cwt. per sq. in. compression, which is satisfactory.

The minus sign is used before the $\frac{W}{A}$ because it is in tension. The net section left at the joints would require to be not less than $\frac{80}{3} = 26{\cdot}\dot{6}$ sq. ins., say 6 ins. by 5 ins., and the bearing area at end of principal rafters by which the tension is put upon the beam should not be less than $\frac{80}{6} = 13{\cdot}\dot{3}$ sq. ins., so that as the rafter is 5 ins. wide it should be let into the tie beam for a depth of, say, 3 ins.

The trusses being 10 ft. centre to centre, the purlins, taking the windward side for maximum load, will have to carry $15 + 18 = 33$ cwts. distributed, then $\frac{bd^2}{10} = 33$, or $bd^2 = 330$; with a depth of 8 ins. the requisite width will be $\frac{330}{8 \times 8} = 5{\cdot}16$, say $5\frac{1}{4}$ ins., but, no doubt, 8 ins. by 5 ins. would be sufficient. For the pole plate the load will be $7{\cdot}5 + 9 = 16{\cdot}5$ cwts. distributed, then $\frac{bd^2}{10} = 16{\cdot}5$, or $bd^2 = 165$; with a depth of 6 ins. the requisite width will be $\frac{165}{6 \times 6} = 4{\cdot}6$, say $4\frac{1}{2}$ ins. For the common rafters the distance apart will be 12 ins., say 14-in. centres, thickness, say, 2 ins., then $\frac{33}{10}$ = weight per foot run on purlin, and multiplying by $\frac{14}{12}$ will give load carried between supports $= \frac{33 \times 14}{10 \times 12} = 3{\cdot}85$ cwts. Then $\frac{2\,d^2}{7{\cdot}2} = 3{\cdot}85$, whence

$$d = \sqrt{\frac{3{\cdot}85 \times 7{\cdot}2}{2}} = \sqrt{13{\cdot}86} = 3{\cdot}72, \text{ say } 4 \text{ ins.}$$

For the ceiling joists at 14-in. centres under a dead load of 11 lbs. per ft. sup. the required size would be $WL = bd^2 = \frac{11}{112} \times \frac{14}{12} \times 10^2 = 11{\cdot}46$, which with a depth of 3 ins. would require a thickness of $\frac{11{\cdot}46}{3 \times 3} = 1{\cdot}27$, say $1\frac{1}{4}$ ins.; this, however, would not allow for the inevitable plumber or bell-hanger

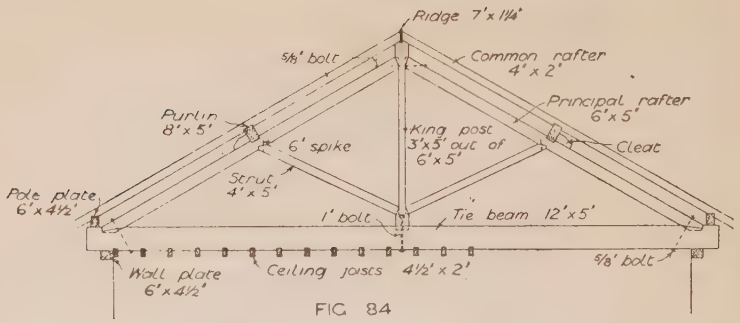


FIG. 84.—Elevation of King Post Truss.

traversing the roof space. To permit of this, allowance must be made for an additional central load of 168 lbs. = 1½ cwts., or an equivalent distributed load of 3 cwts., making the corrected calculation $bd^2 = 11{\cdot}46 + (3 \times 10) = 41{\cdot}46$, and making the depth 4½ ins., the thickness will be $\frac{41{\cdot}46}{4{\cdot}5 \times 4{\cdot}5} = 2{\cdot}04$, say 2 ins.

For the bearing area requisite at each end of the truss we have the load of 24 + 42 = 66 cwts., = 7,392 lbs., then at 250 lbs. per sq. in. and a dimension one way of 5 ins. (= thickness of tie beam) the

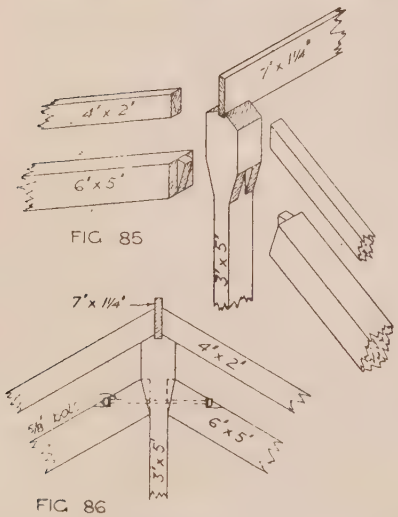


FIG. 85.—Isometric view of Joints at head of King Post.
FIG. 86.—Front elevation of Fig. 85.

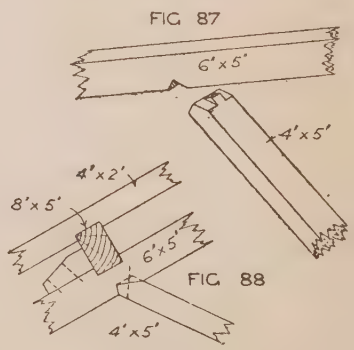


FIG. 87.—Isometric view of Joint between Strut and Principal Rafter.
FIG. 88.—Front elevation of Fig. 87.

other way must be $\frac{7,392}{250 \times 5} = 6$ ins., so that the wall plate should not be of less scantling than, say, 6 ins. by $4\frac{1}{2}$ ins., the same as the pole plate.

The complete elevation of truss may now be drawn out as

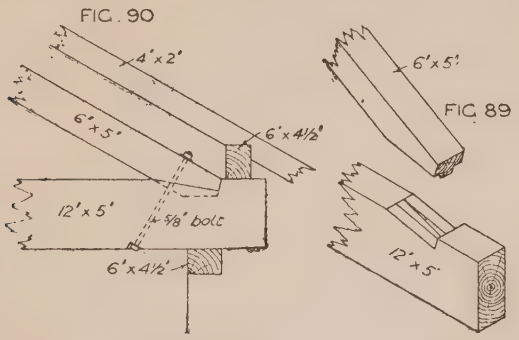


FIG. 89.—Isometric view of Joint at foot of Principal Rafter.
FIG. 90.—Front elevation of Fig. 89.

Fig. 84, and details of the joints as Figs. 85 to 92. Fig. 85 shows an isometric sketch of joints at head of king post, and Fig. 86 front elevation of the same joints; Figs. 87 and 88 show details of joint between strut and principal rafter; Figs. 89 and 90 show details of

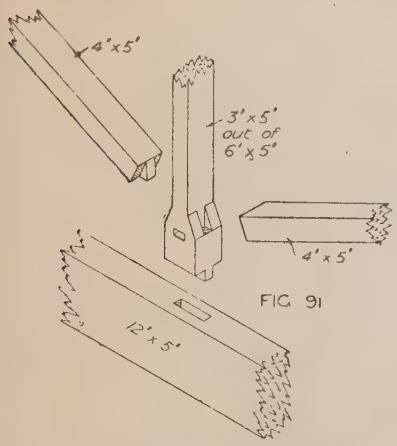


FIG. 91.—Isometric view of Joints at foot of King Post.

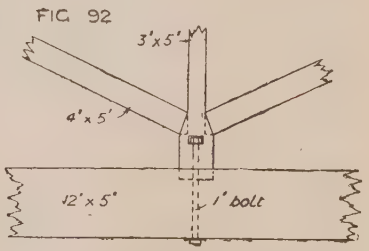


FIG. 92.—Front elevation of Fig. 91.

joint at foot of principal rafter; and Figs. 91 and 92 show details of joints at foot of king post.

CHAPTER VIII

Designing Queen-Post Truss—Explanation of Liability to Deform—Special Stresses Involved—Details of Truss for 36-ft. Span

After determining upon the external shape of a roof, the next step is usually to decide upon the number of points of support required for the common rafters, so that the ordinary scantling of about 4 ins. by 2 ins. may be adopted. This will fix the position of the purlins, and the trusses should be so framed that a strut comes immediately under each purlin to avoid transverse stresses upon the principal rafters. The extreme load on a common rafter is generally somewhere about $\frac{1}{2}$ cwt. per foot run, and, taking this figure for load, the distance L between the supports would be given by the equation $\frac{1}{2} L = \frac{2 \times 4^2}{L}$ whence $L = \sqrt{\frac{2 \times 4^2}{\frac{1}{2}}} = \sqrt{64} = 8$ ft., which will be approximately the maximum distance between the purlins. Then the span is divided up into as many equal parts as will give not more than, say, 8 ft. between purlins. In this way it is seen that a king-post roof truss may be used for spans from, say, 20 ft. to 30 ft., and queen-post trusses for, say, 32 ft. to 46 ft.

The calculation of a queen-post truss brings out some new and interesting features in designing. Owing to the absence of bracing across the central rectangular space, it is a deformable structure, and only keeps its shape by reason of its stiffness; it is, therefore, never seen constructed in metal, because the cross-section of the members would be unsuitable for the special stresses incurred in this form of truss. To exemplify the method of working, a queen-post truss for 36 ft. may be designed, the trusses 12 ft. centre to centre and no ceiling, the finished work being as shown in Fig. 93. The loads upon the

roof will be found as described in the last chapter, and marked upon the frame diagram, Fig. 94; but the exact determination of the loads and stresses in their passage through the common rafters to the truss would be extremely complicated, and dependent to some extent upon the quality of the workman-

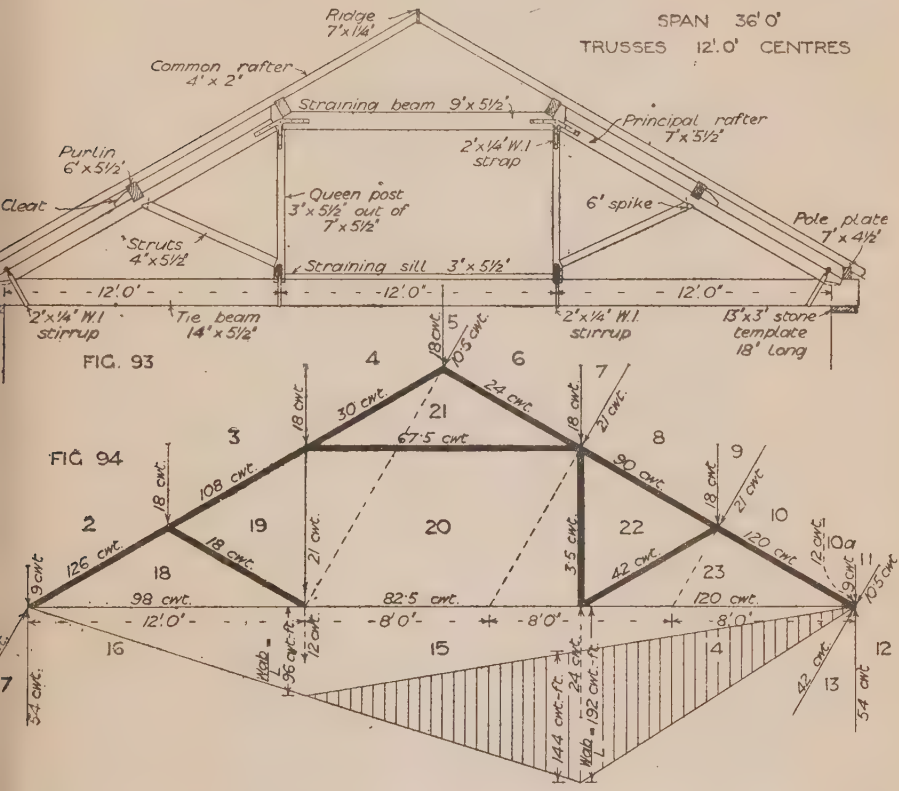


FIG. 93.—Elevation of Queen-Post Truss. FIG. 94.—Frame diagram for Fig. 93.

ship. Adequate information has, however, been given in a previous chapter to enable any one sufficiently interested in the matter to attempt the full application of the principles to the present case. The reactions will be obtained as explained in the last case, and the stress diagram, Fig. 95, worked as follows, but, in order to complete it, the dotted virtual forces must be

introduced. Set down the load line from 1 to 10 and reactions 16-17, 17-1; then 2-18, 16-18; 18-19, 3-19; 4-21, 6-21; 21-20, 19-20; 20-15, 16-15; then 20-22, 8-22; 22-23, 10-23; 23-14, 15-14. The remainder of the load line may now be set down,

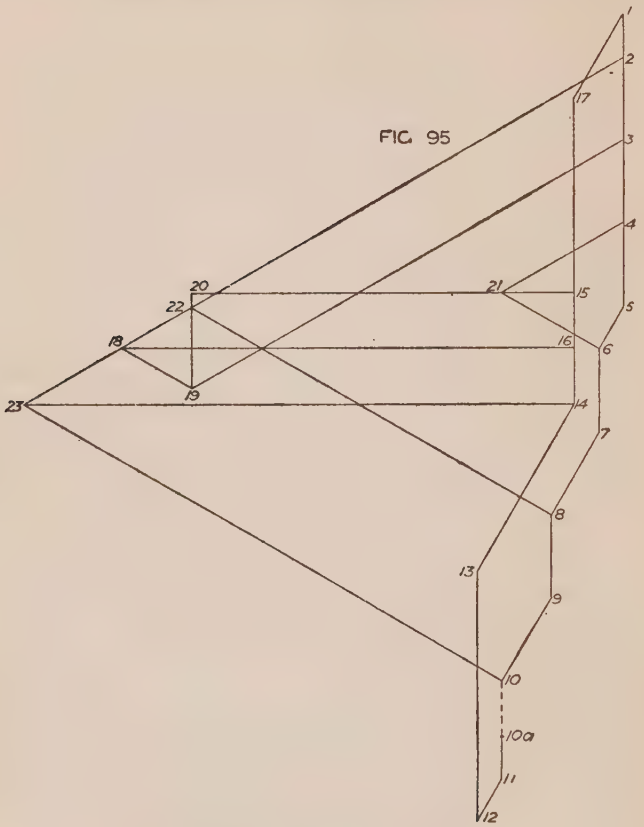


FIG. 95.—Stress diagram for Fig. 94.

working from point 14, and it is found that the virtual force 10-10*a* is required to complete the diagram. The bending moments on the tie beam, which take the place of the virtual forces, will be as shown in Fig. 94. The virtual force at the left-hand queen-post is of opposite character to that at the right-hand queen-post, and each force produces a bending moment diagram in the form of a triangle, with the full span as base.

Subtracting the lesser from the greater leaves the result shown by the shaded diagram in Fig. 94. The calculations are as follows:—

For the principal rafter, with a maximum stress of 126 cwts. and unsupported length of, say, 7 ft., try a 7-in. by $5\frac{1}{2}$-in. section; then by the Rankine-Gordon formula

$$P = \frac{fA}{1 + \frac{1}{ac}\frac{l^2}{r^2}} = \frac{4{,}000 \times 38{\cdot}5}{1 + \frac{1}{2 \times 750}\frac{(7 \times 12)^2}{2{\cdot}5}} = 55{,}000 \text{ lbs.},$$

or 491 cwts., and allowing a factor of safety of 4, the safe load will be $\frac{491}{4} = 123$ cwts.

For the straining beam, with a maximum stress of 67·5 cwts. and unsupported length of 12 ft., try a 9-in. by $5\frac{1}{2}$-in. section; then

$$P = \frac{fA}{1 + \frac{1}{ac}\frac{l^2}{r^2}} = \frac{4{,}000 \times 49{\cdot}5}{1 + \frac{1}{2 \times 750}\frac{(12 \times 12)^2}{2{\cdot}5}} = 30{,}462 \text{ lbs.},$$

or 272 cwts., and $\frac{272}{4} = 68$ cwts. safe load.

For the struts with a maximum compression of 42 cwts. and unsupported length of 7 ft., try a 4-in. by $5\frac{1}{2}$-in. section; then

$$P = \frac{fA}{1 + \frac{1}{ac}\frac{l^2}{r^2}} = \frac{4{,}000 \times 22}{1 + \frac{1}{2 \times 750}\frac{(7 \times 12)^2}{1{\cdot}3}} = 19{,}426 \text{ lbs.},$$

or 173 cwts., and $\frac{173}{4} = 43{\cdot}25$ cwts. safe load.

Instead of finding the radius of gyration from the moment of inertia and sectional area which applies to all forms of section, a special method for rectangular sections may be adopted, viz., $r^2 = \frac{1}{12}d^2$. The accuracy of this is proved as follows:—

$$r^2 = \frac{I}{A} = \frac{\frac{1}{12}bd^3}{bd} = \frac{1}{12}d^2.$$

For the tie beam, with a maximum stress of 120 cwts. and maximum bending moment of 144 cwt.-ft. = 1,728 cwt.-ins., try a 12-in. by $5\frac{1}{2}$-in. section. Then

$$-\frac{W}{A} \pm \frac{M}{Z} = -\frac{120}{66} \pm \frac{1{,}728}{\frac{1}{6}(5\frac{1}{2} \times 12^2)} = -1{\cdot}8 \pm 13 = 14{\cdot}8 \text{ cwts.}$$

per sq. in. tension and 11·2 cwts. per sq. in. compression, which are too high, as the tensile stress should, if possible, be limited to 10 cwts. per sq. in. Try a 14-in. by $5\frac{1}{2}$-in. section; then

$$-\frac{W}{A} \pm \frac{M}{Z} = \frac{120}{77} \pm \frac{1{,}728}{\frac{1}{6}(5\frac{1}{2} \times 14^2)} = 1{\cdot}5 \pm 9{\cdot}6 = 11{\cdot}1 \text{ cwts.}$$

per sq. in. tension and 8·1 cwts. per sq. in. compression. This slightly exceeds the limit laid down, but the scantling is somewhat in excess of that usually adopted in practice.

The queen-posts must be calculated for 21 cwts. tension and 3·5 cwts. compression, as found by stress diagram. Try a 3-in. by $5\frac{1}{2}$-in. section. Then

$$P = \frac{fA}{1 + \frac{1}{ac}\frac{l^2}{r^2}} = \frac{4{,}000 \times 16{\cdot}5}{1 \times \frac{1}{2 \times 750}\frac{(7 \times 12)^2}{0{\cdot}75}} = 9{,}041 \text{ lbs.},$$

or 80 cwts., and $\frac{80}{4} = 20$ cwts. safe load in compression and for tension $\frac{21}{3 \times 5\frac{1}{2}} = 1{\cdot}27$ cwts. per sq. in. The section assumed is therefore in excess of the amount required, but less than that usually adopted in ordinary practice, the difference being due to the allowance made for some carpenters cutting the mortice for the gibs and cotters in the thin part, whereas this mortice should always be cut in the thick part. The straining sill does not need calculation, and may be made 3 ins. by $5\frac{1}{2}$ ins., which is about the size usually adopted. Sometimes cleats on the tie beam against the queen-posts take the place of the straining sill.

The purlins have to carry a distributed load of 39 cwts.; then $W = \frac{bd^2}{L}$ or $bd^2 = 39 \times 12 = 468$, which will be given by a section 9 ins. by $5\frac{1}{2}$ ins. The pole-plate will have a distributed load of 19·5 cwts.; then $bd^2 = 19{\cdot}5 \times 12 = 234$, which will be given by a 7-in. by $4\frac{1}{2}$-in. section. The common rafter will have to carry a distributed load of 3·8 cwts.; then $bd^2 = 3{\cdot}8 \times 7 = 26{\cdot}6$, say 4 ins. by 2 ins. At 250 lbs. per sq. in. the bearing area required on wall plate or stone template will be $\frac{(42 + 54)\ 112}{250} = 43$ sq. ins., $\frac{43}{5{\cdot}5} = 7{\cdot}8$, say 8 ins. minimum length; but the tie beam will run well beyond this, for con-

structive reasons, as shown. If the load on the brickwork is limited to 3 tons per sq. ft., the area of stone template must be not less than $\frac{42 + 54}{3 \times 20} = 1{\cdot}6$ sq. ft., and, with a length of 18 ins., the width will be $\frac{1{\cdot}6}{1{\cdot}5} = 1{\cdot}06$ ft., or, to suit the brickwork, say 13 ins. An isometric sketch of the joints at head of queen-post is shown in Fig. 96, and the front elevation of the same joint in Fig. 97. Figs. 98, 99, and 100 show details of the joints at foot of queen-post. Figs. 101 and 102 show details of

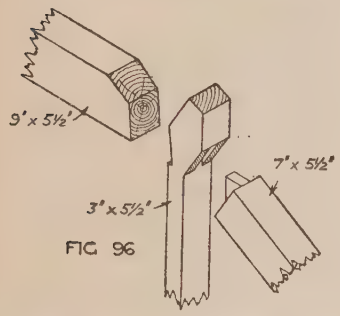


Fig. 96.—Isometric view of Joints at head of Queen-Post.

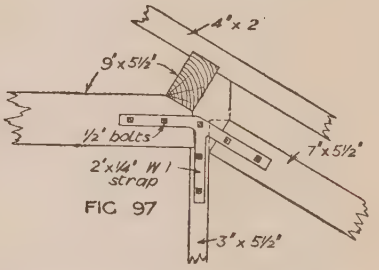


Fig. 97.—Front elevation of Fig. 96.

the joint between strut and principal rafter, and Figs. 103 and 104 show details of the joint between principal rafter and tie-beam.

The portion of the queen-post under the gibs and cotters will be subjected to detrusion or shear along the fibres, and a working stress of 125 lbs. per sq. in. may be allowed for this. Then, as there are two sides to detrude, this figure may be multiplied by $1\frac{1}{2} = 125 \times 1\frac{1}{2} = 187{\cdot}5$ lbs. per sq. in. for double detrusion. Then the least depth required below the mortice will be $\frac{21 \times 112}{187{\cdot}5 \times 5{\cdot}5} = 2{\cdot}28$, say, $2\frac{1}{2}$ ins.; but in the present case we can allow up to $7 - 3\frac{1}{2} = 3\frac{1}{2}$ ins., and this will be a safeguard against splitting by the cotters being too thick.

Having shown how king- and queen-post roof trusses may be precisely calculated, it will be useful to give tables of scantlings that may be adopted without calculation for ordinary cases.

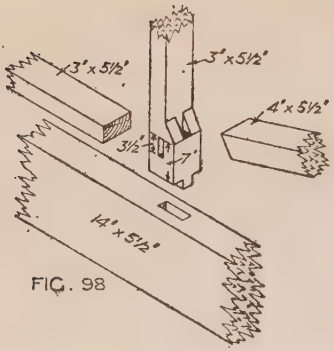


FIG. 98.—Isometric view of Joints at foot of Queen-Post.

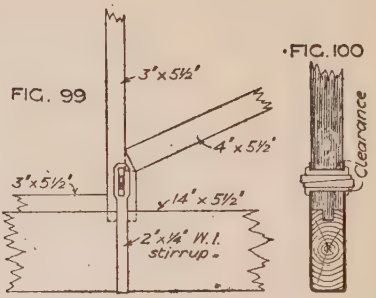


FIG. 99.—Front elevation of Fig. 98.
FIG. 100.—Vertical section of Fig. 99.

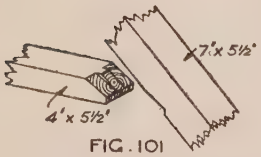


FIG. 101.—Isometric view of Joint between Strut and Principal Rafter.

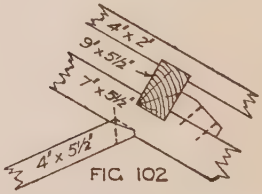


FIG. 102.—Front elevation of Fig. 101.

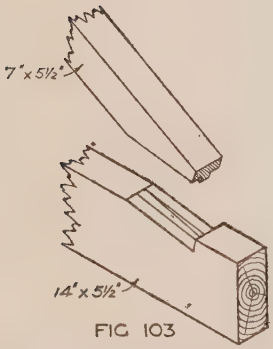


FIG. 103.—Isometric view of Joint between Principal Rafter and Tie Beam.

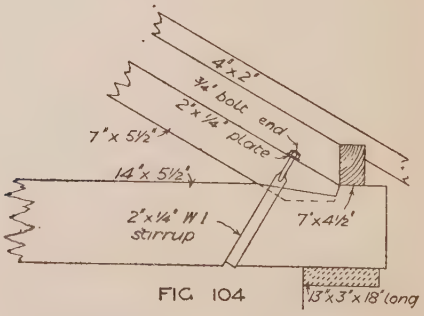


FIG. 104.—Front elevation of Fig. 103.

TABLE OF SCANTLINGS FOR KING-POST TRUSSES.

Baltic fir, 10 ft. apart. Pitch, 25° to 30°. Slated.

Span.	Thickness of Truss and all Members.	Breadth on Elevation.				Purlins.	Common Rafters.
		Tie Beam.	King-Post.	Struts.	Principal Rafters.		
Ft.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.
20	4	8	3	2	4	8 × 4	3½ × 2
22	4½	9	3½	2¼	4	8 × 4	4 × 2
24	5	10	3½	2½	4	8 × 4½	4 × 2
26	5½	11	4	2¾	4	9 × 4½	4½ × 2
28	6	12	4	3	4¼	9 × 5	4½ × 2
30	6½	13	4½	3	4½	9 × 6	5 × 2

Thickness of king-post truss in inches for 10 ft. centres = ¼ span ft. — 1. For other centre distances add or subtract the difference from 10 ft. to or from the span—*e.g.*, 12 ft. 6 ins. centres, 24 ft. span taken as 26 ft. 6 ins. span. Head and foot of king-post, twice width of middle. Reduce thickness of truss by ½ in. and depth of tie beam by 1 in. if there is no ceiling.

TABLE OF SCANTLINGS FOR QUEEN-POST TRUSSES.

Baltic fir, 10 ft. apart. Pitch, 25° to 30°. Slated.

Span.	Thickness of Truss and all Members.	Breadth on Elevation.					Purlins.	Common Rafters.
		Tie Beam.	Queen-Post.	Struts.	Principal Rafters.	Straining Beam.		
Ft.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.
32	4	9	4	2	4½	6	8 × 4	3½ × 2
34	4½	10	4	2	5	6½	8 × 4	4 × 2
36	5	10½	4½	2	5½	7	8 × 4½	4 × 2
38	5½	11	4½	2	5½	7½	8 × 4½	4 × 2
40	5½	11½	5	2½	6	8	9 × 4½	4½ × 2
42	6	12	5	2½	6	8	9 × 4½	4½ × 2
44	6	12½	5½	2½	6½	8½	9 × 5	4½ × 2
46	6	13	6	2½	7	9	9 × 6	5 × 2

CHAPTER IX

Loads on Bressummers—Bending Moments on Continuous Beam—Use of Rolled Joists—Bending Moments and Shearing Forces under Irregular Loading

Structures involving the use of wood have been dealt with first because the material is used principally in rectangular sections, and the study of stresses is somewhat simplified thereby, but the other materials will receive attention in due course.

One of the simplest cases in designing beams occurs in carrying a wall over a shop front. The clear span is usually given to commence with, and the load has to be ascertained. Take the conditions shown in Figs. 105 and 106. For the weight of wall taken as 9 ins. thick allow 1 cwt. per cubic foot, and make no deduction of the window openings, as this will allow for the sashes and frames, sills, plaster, etc. Allow 1 cwt. per foot super for the floor, ceiling, and live load. Allow $\frac{1}{2}$ cwt. per foot super of horizontal measurements for roof truss, covering, wind, ceiling, etc. These are common working allowances for finding load on girder when details are not disclosed. Then the load to be carried will be $17{\cdot}75 \times {\cdot}75 \times 10 \times 1 = 133{\cdot}125$ cwts. for the wall; $17{\cdot}75 \times \frac{14}{2} \times 1 = 124{\cdot}25$ cwts. for the floor; and $17{\cdot}75 \times \frac{14}{2} \times {\cdot}5 = 62{\cdot}125$ cwts. for the roof; in all $133{\cdot}125 + 124{\cdot}25 + 62{\cdot}125 = 319{\cdot}5$ cwts., say 16 tons. By reference to a list of "British Standard Beams," it will be found that on a span of 18 ft. there is a choice between a 12-in. by 6-in. by 54-lb. or a 14-in. by 6-in. by 46-lb. rolled steel joist. The deeper joist is the lighter, and therefore cheaper in itself, but it has the disadvantage of requiring 2 ins. more head room, and is therefore objectionable. Generally a saving in height is worth having, and an alternative arrangement would be to have two shallow joists

side by side. It will be seen from the table that two 10-in. by 5-in. by 30-lb. joists would each carry 8 tons, making together the required total of 16 tons, but the further saving of 2 ins. in depth is obtained at the expense of 60 — 54 = 6 lbs. per foot run more weight than the 12-in. by 6-in. joists, and the 10-in. by 5-in. joists are outside the zig-zag line shown in the table, which

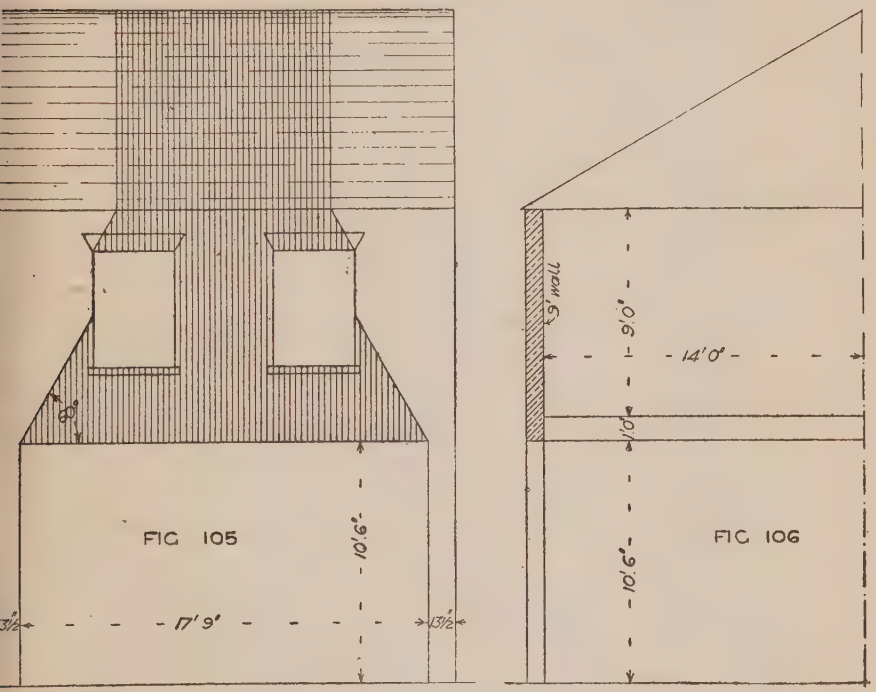


FIG. 105.—Elevation of House, showing Load carried by Girder.
FIG. 106.—Section of Fig. 105.

means that the deflection would be excessive, so that the 12-in. by 6-in. by 54-lb. joist would be the one to adopt. It may be argued that the allowance for weight of floor is excessive, but it will frequently be found that the floor over a shop is occasionally loaded with goods, and it is not safe to make the allowance less. It may also be said that the bonding of the brickwork of the front wall relieves the girder of much of the weight. For example, it may sometimes be sufficient to take the portion

actually carried by the girder as coming within the shaded area on Fig. 105, but it is always better to be on the safe side, and the expense of a little more steel is very slight and well worth the additional security that is obtained.

If the wall had been $1\frac{1}{2}$ bricks thick, the total weight to be carried would have been $319{\cdot}5 + \frac{133{\cdot}125}{2} = 386{\cdot}06$ cwts. $= 19{\cdot}3$, say 20 tons, then two 10-in. by 6-in. by 42-lb. rolled joists would have been suitable, bolted together through the webs with cast-iron separators by the English method, as shown in Fig. 107, or the American method, as in Fig. 108. Girders with separators cost very little more than plain girders, and there is no delay in delivery, whereas if they are connected by a top plate there is always delay in getting the riveting done, and the work is much more expensive. There are, of course, cases in which it is necessary to add plates to rolled joists to obtain the necessary strength, and such cases will be dealt with later on.

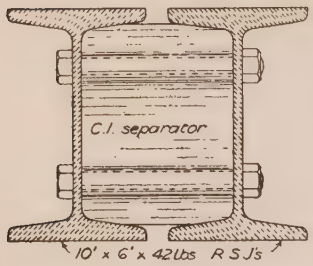


FIG. 107

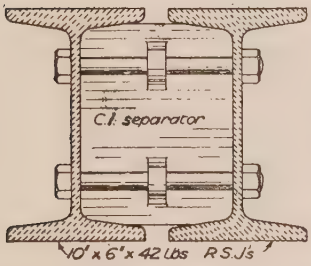


FIG. 108

FIG. 107.—Joists connected by Cast-iron Separators (British method).

FIG. 108.—Joists connected by Cast-iron Separators (American method).

Before leaving this case the supports must be looked to. With a total load of 16 tons there will be 8 tons on each side, and unless the piers are built up in cement not more than 3 tons per sq. ft. should be put upon them. This gives a required bearing area of $\frac{8}{3} = 2\frac{2}{3}$ sq. ft., and the wall being only 9 ins. thick, the length of bearing surface for the stone template under girder will have to be $\frac{2\frac{2}{3}}{.75} = 3{\cdot}55$ ft., which is prohibitive. There are two ways of overcoming the difficulty, either a column may be

put under the centre of the girder, or channel sections may be used as stanchions against the brick piers. Taking the former method, we have the conditions shown in Figs. 58 and 59, p. 42. Now, taking W for the total load and L total length, the maximum bending moment is seen to be $\frac{\frac{1}{2}W \times \frac{1}{2}L}{8} = \frac{\frac{1}{2} \times 16 \times \frac{1}{2} \times 17{\cdot}75}{8}$ $= 8{\cdot}875$ ton-ft. over the column; but the maximum bending moment on the girder without the column is $\frac{WL}{8} = \frac{16 \times 17{\cdot}75}{8}$ $= 35{\cdot}5$ ton-ft., or four times as much, so that it is clear that a very much lighter section may be used for the girder with the

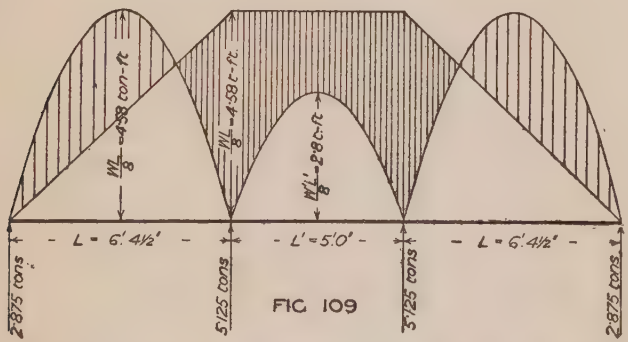


FIG 109

FIG. 109.—Bending moment diagram for Girder with three Spans.

column. To find this section from the table we require to know the distributed load over the half length that will give a bending moment of 8·875 ton-ft., that is $\frac{WL}{8} = 8{\cdot}875$, but $L = \frac{17{\cdot}75}{2}$ $= 8{\cdot}875$, therefore $W = \frac{8{\cdot}875 \times 8}{8{\cdot}875} = 8$ tons, and to carry 8 tons over a span of 8·875, say 9 ft., will require a joist 8 ins. by 4 ins. by 18 lbs., or two 6 ins. by 3 ins. by 12 lbs., which most builders would prefer to use. If the shop has a central doorway, it will necessitate two columns, say 5 ft. centre to centre. The bending moment diagram would then be approximately as Fig. 109, and the load on the centre span will be $\frac{5}{17{\cdot}75} \times 16 = 4{\cdot}5$ tons, while that on the side spans will be

$\frac{16 - 4{\cdot}5}{2} = 5{\cdot}75$ tons. The load on the columns may be taken approximately as half the load on each adjoining span as marked, bringing the maximum bending moment down to

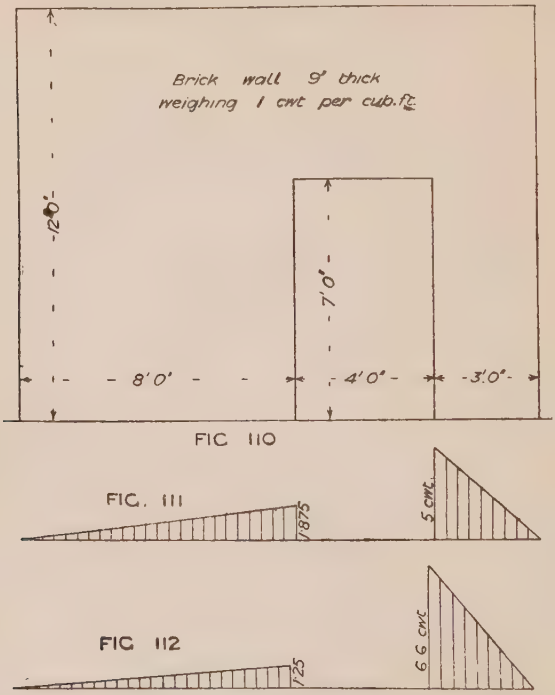


Fig. 110.—Elevation of Brick Partition to be carried by Girder.
Figs. 111 and 112.—Methods of dividing Load over doorway on to the sides.

$\frac{WL}{8} = \frac{5{\cdot}75 \times 6{\cdot}375}{8} = 4{\cdot}58$ ton-ft., equivalent to a distributed load over a span of 6·375 ft. of $\frac{4{\cdot}58 \times 8}{6{\cdot}375} = 5{\cdot}75$ tons, for which a pair of 4-in. by 3-in. by 9½-lb. rolled steel joists will be sufficient.

The next case to be taken is to find the section required to carry a brick partition with a 4-ft. door opening 3 ft. from one side over a clear span of 15 ft., as shown in Fig. 110. The load over the doorway is transmitted down the two sides, but the

exact distribution of the loading is somewhat doubtful. There are two possible views: the simpler one is that half the load goes each way, and that it is divided over the intervening space in regular proportion from maximum to nothing. Thus, load over door $= 4 \times 5 \times \cdot75 \times 1 = 15$ cwts., half each way $= 7{\cdot}5$ cwts.; then for left side $\frac{7{\cdot}5}{8} = \cdot9375$ per foot run, and $\cdot9375 \times 2 = 1{\cdot}875$ cwts. increase of load at door jamb, as shown in Fig. 111. In a similar manner the extra load on the right will be $\frac{7{\cdot}5 \times 2}{3} = 5$ cwts. at door jamb. The other view is that the load over doorway is transmitted to each abutment in inverse proportion to the distance from its centre of gravity, and divided over the intervening space as before, viz., on the left $\frac{2+3}{15} \times 15 \times \frac{2}{8} = 1{\cdot}25$ cwts., and on the right $\frac{2+8}{15} \times 15 \times \frac{2}{3} = 6{\cdot}6$ cwts., as shown in Fig. 112. As the former view gives the greater bending moment on the girder, it will be best to adopt it; the weight of the wall is $12 \times \cdot75 \times 1 = 9$ cwts. per foot run, then the load diagram will be as shown in Fig. 113. It will now be necessary to find the maximum bending moment shown in Fig. 114, proceeding as follows: Taking the load on left-hand side as a concentrated load acting at the centre of gravity, the bending moment under the load will be $\frac{Wab}{L} = \frac{79{\cdot}5 \times 4{\cdot}1 \times 10{\cdot}9}{15} = 237$ cwt.-ft. shown by the triangle *a c b*, which will be the bending moment diagram taking the load as concentrated. As the load is not concentrated, a perpendicular must be dropped from the end of the load cutting *b c* in *d*, giving the triangle *a d b* as part of the complete diagram. Similarly, the right-hand load will give a final triangle *a f b*, and then the triangles *a d b* and *a f b* must be added together, giving the figure *a g h b*. Then to complete the diagram it will be sufficiently approximate to assume the load on left-hand side as evenly distributed over the span of 8 ft., giving a maximum bending moment of $\frac{WL}{8} = \frac{79{\cdot}5 \times 8}{8} = 79{\cdot}5$ cwt.-ft., which must be set off opposite the centre of gravity of

the load, the diagram being as shown by the two semi-parabolas *a j* and *j g* which are constructed as in Fig. 24, p. 18. In a similar manner the semi-parabolas *b k* and *k h* must be added to the right-hand side, the complete diagram being *a j g h k b*. By

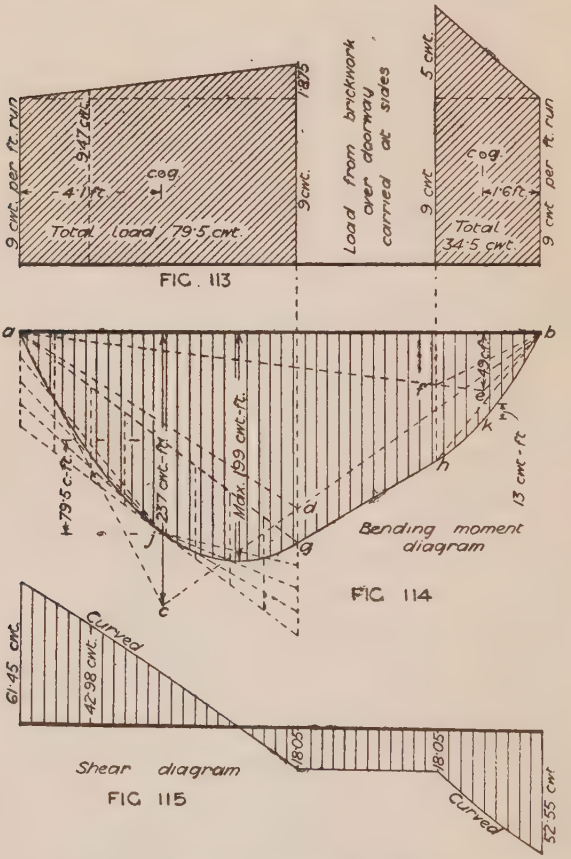


FIG. 113.—Load diagram for Fig. 110.
FIG. 114.—Bending moment diagram for Fig. 113.
FIG. 115.—Shear diagram for Fig. 113.

scaling, the maximum bending moment (M) is found to be 199 cwt.-ft. = say, 10 ton-ft., then $M = \frac{WL}{8}$ or $W = \frac{8\,M}{L} = \frac{8 \times 10}{15} = 5{\cdot}3$ tons equivalent distributed load over the 15 ft.

span, requiring a 9-in. by 4-in. by 21-lb. rolled steel joist, or two 7-in. by 4-in. by 16-lb. rolled steel joists. The reaction at left-hand abutment $= \frac{10{\cdot}9}{15} \times 79{\cdot}5 + \frac{1{\cdot}6}{15} \times 34{\cdot}5 = 61{\cdot}45$ cwts., and at the right-hand abutment $\frac{13{\cdot}4}{15} \times 34{\cdot}5 + \frac{4{\cdot}1}{15} \times 79{\cdot}5 = 52{\cdot}55$, the two together making up the total load of $79{\cdot}5 + 34{\cdot}5 = 114$ cwts. To construct the shear diagram, set up the left-hand reaction above the beam, which is positive, as in Fig. 115, then the shear at any point is equal to the reaction less the load between the abutment and the point under consideration, thus at 2 ft. from left abutment the shear $= 61{\cdot}45 - \left(\frac{9 + 9{\cdot}47}{2} \times 2\right) = 42{\cdot}98$ cwts. and so on. At the end of the left-hand load the shear will be $61{\cdot}45 - 79{\cdot}5 = 18{\cdot}05$ cwts. negative, and will be set off below the horizontal line. From this point to the beginning of the right-hand load the shear will be constant at 18·05 cwts., and will then increase till it reaches the right-hand abutment, where the shear $= 18{\cdot}05 + 34{\cdot}5 = 52{\cdot}55$ cwts. negative.

Allowing a safe load of 12 tons per sq. ft. on the York stone template, the two joists will require a bearing area of $\frac{61{\cdot}45}{20 \times 12} = 0{\cdot}256$ sq. ft., and as the combined width is 8 ins., the joists must rest on the stone for at least 4·6 ins., say 6 ins. Allowing 3 tons per sq. ft. on the brickwork, the area of stone required $= \frac{61{\cdot}45}{20 \times 3} = 1{\cdot}02$ sq. ft., and assuming the width to be 9 ins., the length will be $\frac{1{\cdot}02}{0{\cdot}75} = 1{\cdot}36$ ft., say 18 ins.

CHAPTER X

Finding Strength of Rolled Joist used as Stanchion—Moment of Inertia—Section Modulus—Safe Loads on Materials—Reinforced Concrete Foundation

Keeping for the present to the more elementary parts of practical design, we now proceed to show how to find the strength of a rolled steel joist used as a stanchion. Assume a British Standard Beam (B.S.B., No. 14), 8 ins. by 6 ins. by 35 lbs. per foot run, 10 ft. high, with fixed ends; the safe load on this as a stanchion is given in Dorman, Long & Co.'s tables as 47 tons, and the section is shown in Fig. 116. There are two axes, $X - X$ and $Y - Y$; the moments of inertia are given in the tables as 110·5 about $X - X$, and 17·95 about $Y - Y$, the radii of gyration as 3·28 and 1·32, the section moduli 27·62 and 5·98, and the sectional area 10·29 sq. ins., but all tables do not agree. The moments of inertia can be obtained graphically as follows. For the moment of inertia about the axis $X - X$, divide the section horizontally into any convenient number of parts. Find the area and centre of gravity of each part, and from each centre of gravity draw a horizontal line and number the spaces as shown. Set down the area of the fourteen parts as a load line, marked 1 to 15, in Fig. 117, and fix pole 0 at right angles, so that 1 — 0 is equal to half 1 — 15. Join 0 to the points 1, 2, 3, etc., on the load line, and parallel with these vectors, across their respective spaces, draw lines to construct the inertia area Fig. 118. As a check upon the work the intersection of the lines across spaces 1 and 15, parallel with 0 — 1 and 0 — 15, should be on the axis $X - X$, it being noted that space 1 is the space between line 1 — 2 anti-clockwise round to the neutral axis on the further side of the inertia area diagram, and space 15 is similarly the clockwise space from line 14 — 15 to the neutral axis. The next step is to measure the areas of the

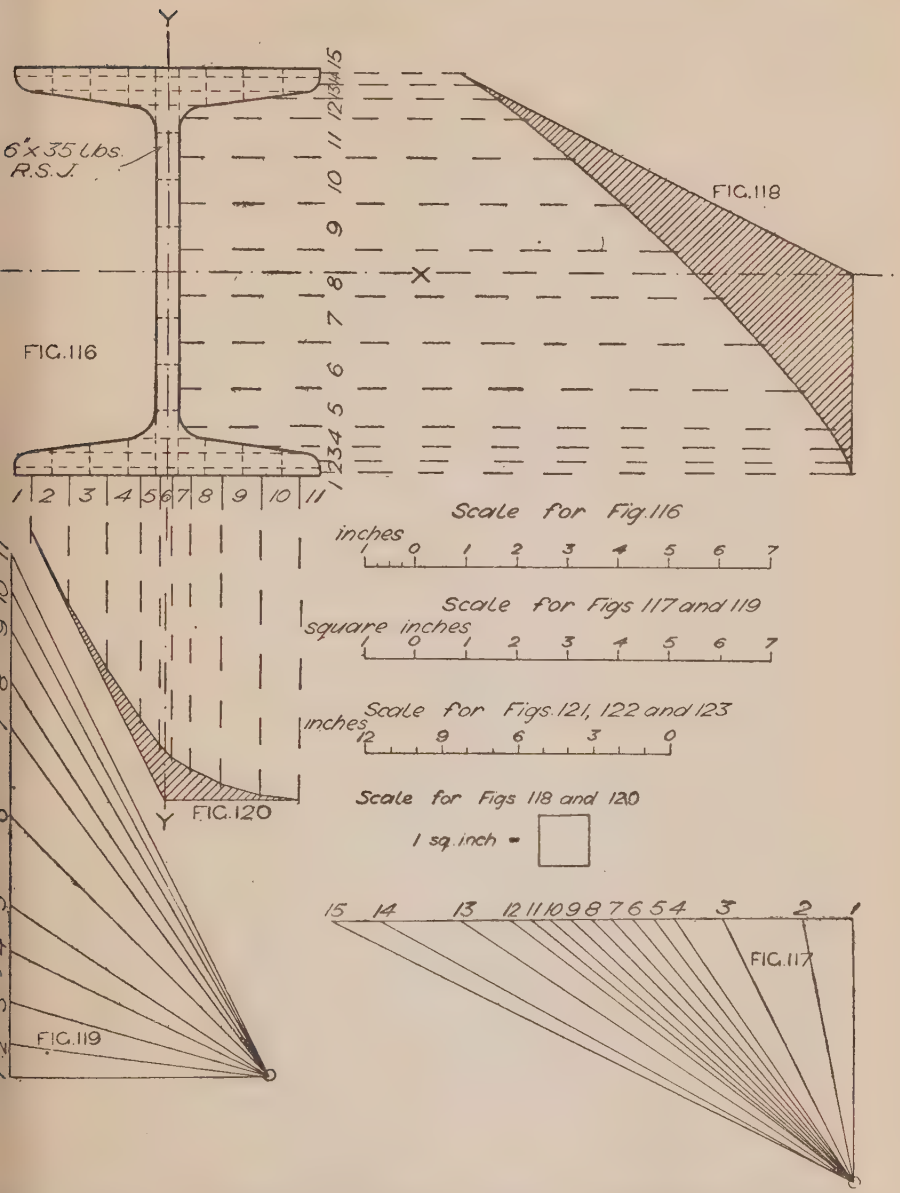


FIG. 116.—Section of Standard Rolled Steel Joist. FIG. 117.—Polar diagram for moment of inertia on $X - X$. FIG. 118.—Inertia area for moment of inertia on $X - X$. FIG. 119.—Polar diagram for moment of inertia on $Y - Y$. FIG. 120.—Inertia area for moment of inertia on $Y - Y$.

joist section and inertia area by planimeter. Then the area of the joist is found to be 2·576 sq. ins., but as the joist was drawn out one-half full size, the true area $= 2{\cdot}576 \times 2^2 = 10{\cdot}3$ sq. ins., and similarly as the inertia area $= 2{\cdot}67$ sq. ins., the true area $= 2{\cdot}67 \times 2^2 = 10{\cdot}68$. The moment of inertia (I) = the area of section (A) multiplied by the inertia area (a), or $I = Aa = 10{\cdot}3 \times 10{\cdot}68 = 110{\cdot}0$, about axis $X - X$, as against 110·5 given by the tables, and it should be noted that the greater the number of divisions taken on the section the nearer will be the result to that given by standard tables. In the same way the moment of inertia about $Y - Y$ may be found as shown by Figs. 119 and 120, then $I = Aa = 10{\cdot}3 \times 1{\cdot}74 = 17{\cdot}92$, against 17·95 as given by the tables. The moments of inertia and section moduli are not required for use when the load is axial, but they will be wanted later on. Using the Rankine-Gordon formula, as given at p. 50, and taking the least radius of gyration

$$P = \frac{fA}{1 + \left(\frac{1}{a\ c}\right)\frac{l^2}{r^2}} = \frac{40{,}000 \times 10{\cdot}29}{1 + \frac{1}{4 \times 9{,}000}\left(\frac{10 \times 12}{1{\cdot}32}\right)^2} = \frac{411{,}600}{1{\cdot}23}$$

$= 334{,}634$ lbs. ultimate load, and allowing a factor of safety of 4, the safe load $= \frac{334{,}634}{4} = 83{,}658{\cdot}5$ lbs., or 37·35 tons, against 47 tons as given by Dorman, Long & Co.'s tables, and 35 tons by Edward Wood & Co.'s tables.

Another method of estimating the strength of a stanchion is to allow a certain stress per sq. in., according to the ratio of $\frac{l}{r}$, that is, length to least radius of gyration. This is adopted in the new Building Act Amendments (1909) of the London County Council, as shown in table on p. 83.

The last column is the most useful, and it will be seen that up to the ratio of 160 the safe load can be memorised by the formula $6{\cdot}5 - \frac{1}{40}\left(\frac{l}{r}\right)$. By this table, taking the ratio in the present case as $\frac{l}{r} = \frac{10 \times 12}{1{\cdot}32} = 91$, then $6{\cdot}5 - \frac{91}{40} = 4{\cdot}225$ tons per sq. in. safe load. Sectional area $= 10{\cdot}29$, then safe load $= 10{\cdot}29 \times 4{\cdot}225 = 43{\cdot}47$ tons, which may be taken as the

L.C.C. safe load. These different results, 37·35 tons, 47 tons, 35 tons, and 43·47 tons, are not satisfactory, as one naturally has some doubt what to do in such circumstances. It is, however, a usual experience, the more formulæ that are tried the greater the number of differences produced. The L.C.C. safe load may be worked to.

Safe Load on Mild Steel Pillars.

Ratio of Length to Least Radius of Gyration.	Working Stresses in tons per sq. in. of Section.		
	Hinged Ends.	One End Hinged and One End Fixed.	Both Ends Fixed.
20	4·0	5·0	6·0
40	3·5	4·5	5·5
60	3·0	4·0	5·0
80	2·5	3·5	4·5
100	2·0	3·0	4·0
120	1·0	2·5	3·5
140	0·0	2·0	3·0
160	—	1·0	2·5
180	—	0·0	1·5
200	—	—	0·5
210	—	—	0·0

If the stanchion be so fixed that it cannot bend in the direction of the least radius of gyration, it may be permissible to calculate its strength upon the basis of the greatest radius of gyration, but special sanction must be obtained for such structures when erected in London. The 8-in. by 6-in. by 35-lb. rolled steel joist will then have a carrying capacity of

$$P = \frac{fA}{1 + \frac{1}{a}\frac{1}{c}\left(\frac{l}{r}\right)^2} = \frac{40{,}000 \times 10{\cdot}29}{1 + \frac{1}{4 \times 9{,}000}\left(\frac{10 \times 12}{3{\cdot}28}\right)^2} = 396{,}914 \text{ lbs.}$$

ultimate load, or $\frac{396{,}914}{4} = 99{,}228{\cdot}5$ lbs. or 44·3 tons, and it must be clearly understood that this load is a central dead load, not applied by means of a girder attached to either web or flange.

The next step will be to consider the size and construction of the base of the stanchion. Before this can be determined the safe load upon various kinds of foundation must be agreed upon. The following table gives the safe load on materials supporting the ends of girders, stanchions, etc.

SAFE LOAD ON MATERIALS.

Material.	Ordinary Load. Tons per Foot Super.	Proportion of Fracture Load.	Maximum Load. Tons per Foot Super.
Granite	20	$\frac{1}{15}$	30
Portland and compact limestone	15	$\frac{1}{15}$	20
Hard York stone . .	12	$\frac{1}{10}$	15
Limestone (ordinary) . .	6	$\frac{1}{10}$	6
Blue brick in cement . .	9	$\frac{1}{8}$	12
Stock „ „ . .	6	$\frac{1}{5}$	8
„ „ lias mortar .	5	$\frac{1}{4}$	6
„ „ grey lime mortar	3	$\frac{1}{4}$	4
Portland cement concrete (6 to 1)	5	$\frac{1}{8}$	10
Lias lime concrete (4 to 1)	3	$\frac{1}{6}$	5
Gravel and natural compact earth . . .	2	—	—
Made ground rammed in layers	1	—	—

The safe load on the cementing materials may be taken as follows :—

Grey or stone lime mortar—

50 lbs. per sq. in. or 3 tons per sq. ft. compression.

25 lbs. per sq. in. or $1\frac{1}{2}$ tons per sq. ft. tension.

Lias lime mortar—

150 lbs. per sq. in. or 9 tons per sq. ft. compression.

50 lbs. per sq. in. or 3 tons per sq. ft. tension.

Portland cement mortar—

200 lbs. per sq. in. or 12 tons per sq. ft. compression.

75 lbs. per sq. in. or 5 tons per sq. ft. tension.

Pressed gault, Fletton and Leicester red bricks are inter-

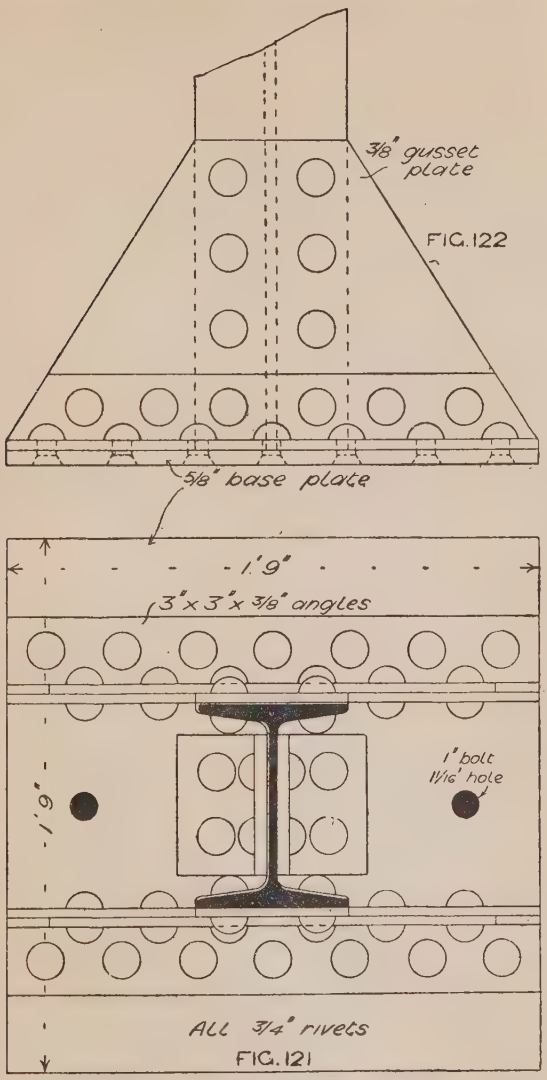


FIG. 121.—Plan of Base of Stanchion.
FIG. 122.—Front elevation of Base of Stanchion.

mediate in strength between London stocks and Staffordshire blue bricks. In architectural work, with dead loads only, the figures in the last column may be worked to, but where vibration

occurs, the factor of safety allowed by the previous column is not too great.

Assuming the stanchion to rest upon a thick York stone base capable of bearing a safe load of 15 tons per square foot, the area of the base must be $\frac{43{\cdot}47}{15} = 2{\cdot}9$ sq. ft. $= 1{\cdot}7$ ft. square, say 1 ft. 9 in. The plan will then be as in Fig. 121, front elevation Fig. 122, and side elevation, Fig. 123. The thickness of base

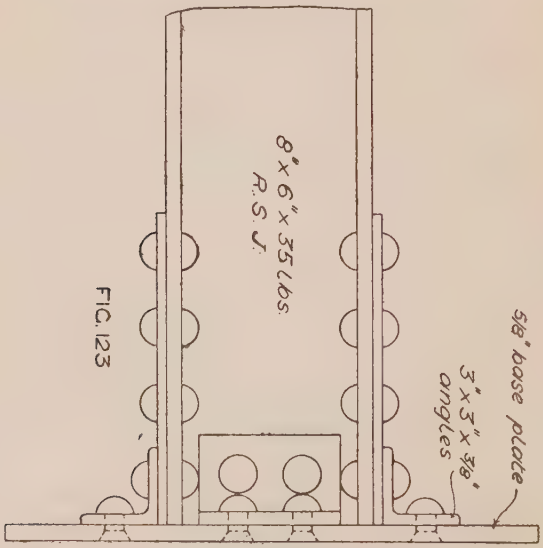


FIG. 123.—Side elevation of Base of Stanchion.

plate depends chiefly upon the unsupported projecting portions, and may be determined by the useful empirical rule, thickness $= \frac{1}{5}$ projection, which in the present case $= \frac{3{\cdot}125}{5} = 0{\cdot}625$ ins. or $\frac{5}{8}$ ins. This can be checked as follows: Taking the projecting portion as a cantilever with an evenly distributed upward pressure in the nature of reaction of $\frac{15 \text{ tons per sq. ft.}}{12 \times 12} = 0{\cdot}104$ ton per sq. in. Then $\frac{WL^2}{2} = ZC$, whence $C = \frac{WL^2}{2Z} = \frac{0{\cdot}104 \times 3{\cdot}125^2}{2 \times \frac{1}{6}(1 \times 0{\cdot}625^2)} = 7\frac{1}{2}$ tons per sq. in. The number of rivets

to take the shear will be found as follows: The area of a $\frac{3}{4}$-in. rivet = 0·44 sq. in., and allowing 5 tons per sq. in. single shear, each rivet will take 0·44 × 5 = 2·2 tons shear, then $\frac{43 \cdot 47}{2 \cdot 2}$ = say, 20 rivets through stanchion. This will be sufficiently obtained by 8 rivets through each flange, and two others in double shear through the web. The thickness of gusset plate should be not more than the thickness of stanchion web, and not less than $\frac{2}{3}$ of the thickness, in this case, say, $\frac{3}{8}$ in. If the stanchion could be always relied upon to bear evenly over the base plate, the riveting might be reduced, but it is usual to provide sufficient sectional area of rivets in shear to take the whole load.

The relative cost for equal bulk of foundation work in different materials may be taken as follows:—

	Per Cub. Ft.	Comparison.
	s. *d.*	
York stone and labour .	6 0	12
Stock brickwork in cement .	1 6	3
Cement concrete 1 to 6 .	0 6	1

This great difference in value and the keen competition to keep down cost leads to the omission of the stone and brickwork, but in practice the difference in cost does not come out so great as shown, owing to the reduction in size of space occupied. For example, let the York stone project 6 ins. all round the base plate and be 1 ft. 6 ins. thick, the contents will be 2·75 × 2·75 × 1·5 = 9·84, say, 10 cub. ft. at 6*s.* = £3 for 18 ins. deep, or £2 per foot of depth. Then, say, brickwork 18 ins. deep and 3 ft. average width of side giving 3 × 3 × 1·5 = 13·5 cub. ft. 1*s.* 6*d.* = £1 0*s.* 3*d.*, or 13*s.* 6*d.* per foot of depth. Then the concrete, say, 5·5 × 5·5 × say, 3 ft. deep = 90·75 cub. ft. at 6*d.* = £2 5*s.* 4½*d.*, or 15*s.* 1½*d.* per foot of depth. The relative cost, taking into account the space occupied, will, therefore, be 40*s.*, 13*s.* 6*d.*, and 15*s.* 1½*d.*, and the relative values about 3, 1, and 1⅛.

It may be desirable to carry the stanchion by a reinforced concrete base spread over the soil. When concrete is made by specialists, it may be relied upon to carry double the load that could be put upon it in ordinary circumstances. With British Standard Cement and skilled labour, the following loads may be put upon cement concrete.

SAFE LOAD ON STANDARD CONCRETE TWELVE MONTHS OLD.

Composition.			Safe Load. Tons per sq. ft.	
P.C.	Sand.	Ballast.	Safety Factor 2.	Safety Factor 4.
1	1	1	50	25
1	1	2	39	20
1	1½	3	32½	16
1	2	4	28½	14
1	2½	5	25	12½
1	3	6	22½	11
1	3½	7	20	10
1	4	8	17½	9
1	4½	9	15	7½
1	5	10	12½	6

REDUCTIONS OF STRENGTH FROM ABOVE TABLE IN CONCRETE NOT MATURED.

Age 14 days	reduction	75	per cent.
21 „	„	50	„
1 month	„	35	„
3 „	„	15	„
6 „	„	7½	„
9 „	„	2½	„

The significance of this is that concrete work may be quite safe with the structural load during construction, and by the time it is desired to load up the building the concrete will have matured sufficiently to take the desired external load.

Assume the soil capable of carrying, say, $1\frac{1}{2}$ tons per sq. ft., the load 43·47, say 44 tons, the area required $= \frac{44}{1\frac{1}{2}} = 29$ sq. ft.,

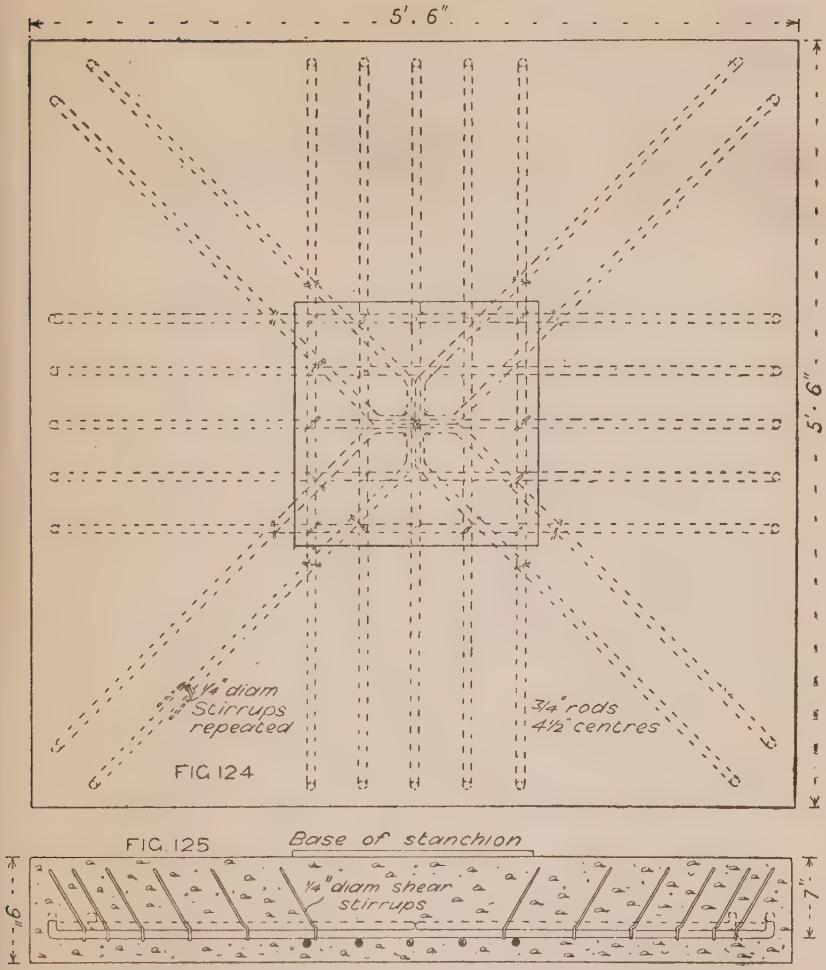


FIG. 124.—Plan of Reinforced Concrete Foundation for Stanchion.
FIG. 125.—Section of Fig. 124.

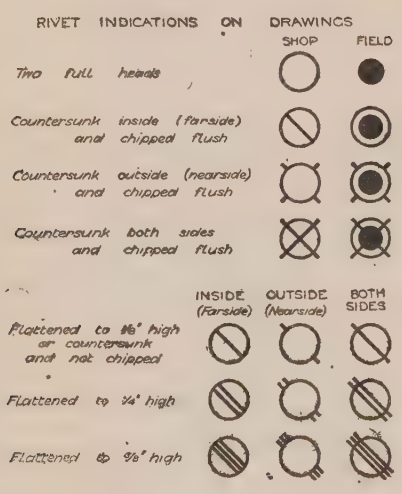
say 5 ft. 6 ins. square. The projecting portion must be taken as a cantilever, and the load on 1 ft. run will be $1{\cdot}875 \times 1 \times 1{\cdot}5 = 2{\cdot}8125$ tons, or 56·25 cwts. distributed. A handy formula for the approximate strength of a reinforced concrete beam is $W = (\cdot 37\ p + \cdot 214)\ \frac{bd^2}{L}$, where W = safe load in cwts.

distributed, including weight of beam, p = percentage of reinforcement, b = breadth in inches, d = depth to centre of reinforcement in inches, L = span in feet. This formula is for beams supported at both ends with a maximum bending moment of $\frac{WL}{8}$, but for a cantilever with a distributed load, the maximum bending moment $= \frac{WL}{2}$, or four times that of the beam. Therefore a cantilever will carry only one-fourth of the load that a beam of similar section can take, so that the formula may be put down as $W = (\cdot 37\, p + \cdot 214)\, \frac{bd^2}{4L}$. Allowing a reinforcement of $1\frac{1}{2}$ per cent., $56 \cdot 25 = (\cdot 37 \times 1\frac{1}{2} + \cdot 214)\, \frac{12\, d^2}{4 \times 1 \cdot 875}$, whence $d^2 = \frac{56 \cdot 25 \times 4 \times 1 \cdot 875}{12\, (\cdot 37 \times 1\frac{1}{2} + \cdot 214)} = 45 \cdot 72$, or $d = \sqrt{45 \cdot 72} = 6 \cdot 76$, say 7 ins. to centre of reinforcement. The area of steel required $= \frac{12 \times 7 \times 1\frac{1}{2}}{100} = 1 \cdot 26$ sq. ins., which will be given by $\frac{3}{4}$-in. rods, $4\frac{1}{2}$ ins. centre to centre, as shown in the plan Fig. 124 and section Fig. 125.

CHAPTER XI

Riveting—Strength of Rivets and Bolts—Designing Connections between Girders and Stanchions—Grillage Foundation

Very few of the stanchions that are used in a building can be said to have axial loads. Even when a stanchion occurs in the middle of a floor girder the girder may be loaded on one side of the stanchion, and not on the other, so that it is necessary now



to consider the effect of a side load. Assume the case shown in Figs. 126 and 127 where a 14-in. by 6-in. by 57-lb. rolled joist brings a load of 10 tons on one side of a stanchion, the unsupported length of which is assumed to be 12 ft., being fixed at girder level by supports not shown. There is also an axial load of 10 tons assumed to come from roof or otherwise. The connections may be designed first. The cleats are usually riveted to the girder and bolted to the stanchion, except in steel

framed buildings, when it is often specified that all connections shall be riveted throughout. Those in the girder are then known as "shop" rivets, and those in the stanchion as "field" rivets. Rivets are shown on drawings by different signs to indicate the various conditions, as shown in the diagram on p. 91.

The safe loads on rivets and bolts may be taken as in the following tables:—

Safe Load on Mild Steel Rivets.

Diameter in Inches.	Direct Tension at 6½ tons per sq. in.	Single Shear at 5 tons per sq. in.	Double Shear at 7½ tons per sq. in.
⅝	2·0	1·5	2·3
¾	2·9	2·2	3·3
⅞	3·9	3·0	4·5
1	5·1	3·9	5·9
1⅛	6·5	5·0	7·5

The diameter taken is that of the hole for rivet.

Safe Load in Shear on Wrought Iron Bolts.

Diameter in Inches.	Ordinary at 3 Tons per Sq. In.	Driving Fit at 4 Tons per Sq. In.
⅝	0·9	1·2
¾	1·3	1·8
⅞	1·8	2·4
1	2·3	3·1
1⅛	3·0	4·0
1¼	3·7	4·9

It is necessary to consider the bearing pressure of a rivet or bolt in its hole, when the parts held are comparatively thin, the bearing area being measured by the diameter × thickness. This is in order that the side pressure, when the rivet is under shear stress, may not crush the rivet or the plate. The following table gives the value of various diameters of rivets and bolts, and

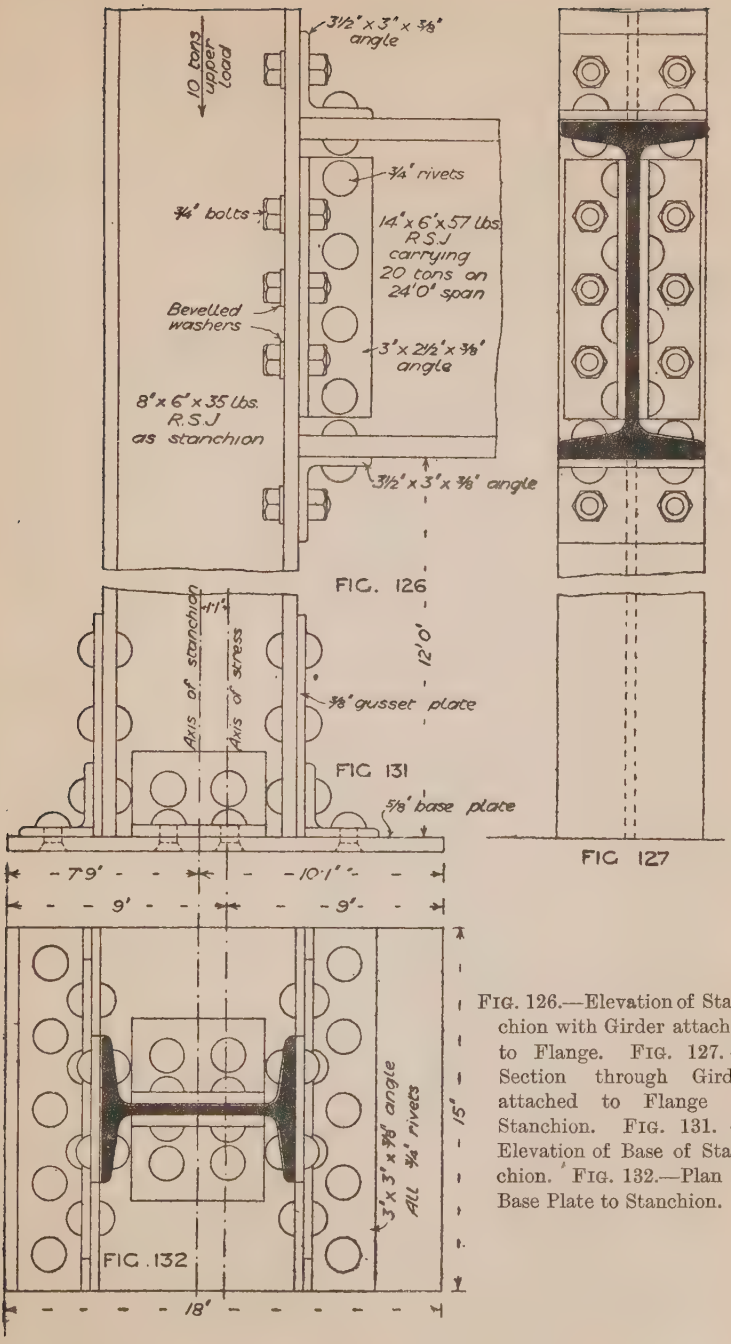


FIG. 126.—Elevation of Stanchion with Girder attached to Flange. FIG. 127.—Section through Girder attached to Flange of Stanchion. FIG. 131.—Elevation of Base of Stanchion. FIG. 132.—Plan of Base Plate to Stanchion.

thicknesses of plates, the thinner plate being taken as the measure when there is any difference:—

BEARING PRESSURES ON WROUGHT-IRON BOLTS AT 6 TONS PER SQ. IN.

Diameter of Bolt in Inches.	Thickness of Plate in Inches.					
	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$
$\frac{5}{8}$	0·94	1·17	1·41	1·64	1·87	2·11
$\frac{3}{4}$	1·12	1·40	1·69	1·97	2·25	2·53
$\frac{7}{8}$	1·31	1·64	1·97	2·29	2·62	2·95
1	1·50	1·87	2·25	2·62	3·00	3·37
$1\frac{1}{8}$	1·69	2·11	2·53	2·95	3·37	3·79
$1\frac{1}{4}$	1·87	2·34	2·81	3·28	3·75	4·22

BEARING PRESSURES ON MILD STEEL RIVETS AT 10 TONS PER SQ. IN.

Diameter of Rivet in inches.	Thickness of Plate in inches.					
	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$
$\frac{5}{8}$	1·56	1·95	2·34	2·73	3·12	3·51
$\frac{3}{4}$	1·87	2·34	2·81	3·28	3·75	4·22
$\frac{7}{8}$	2·19	2·73	3·28	3·83	4·37	4·92
1	2·50	3·12	3·75	4·37	5·00	5·62
$1\frac{1}{8}$	2·81	3·51	4·22	4·92	5·62	6·32

When the bearing value is less than the shear value the former must be taken instead of the latter.

It is usual to keep to $\frac{3}{4}$-in. diameter as much as possible, as the best work is made with hand riveting in this size. The load from end of girder being 10 tons, the number of $\frac{3}{4}$-in. bolts required to resist shear will be $\frac{10}{1{\cdot}3} = 7{\cdot}7$, say, 8 bolts, and generally it is considered well to have half the required number

below the girder, but there is no necessity for this. To get two rows of bolts in one flange of an angle cleat, it must have a clear width on the inside of not less than $4\frac{1}{2}$ times the diameter of bolt, the lower angle cleat would, therefore, be $\frac{3}{4} \times 4\frac{1}{2} = 3\frac{3}{8}$ ins. in the clear, which would be a 4-in. by 3-in. by $\frac{7}{16}$-in. angle. In this case, however, there will be 8 bolts easily placed without counting the bottom cleat, and although the top cleat can be omitted, it is very useful to stiffen the connection so that a single row of bolts may be used in the bottom cleat, and the width of angle to enable the nuts to clear the rivet heads at right angles will be $3\frac{1}{2}$ ins., making the cleat $3\frac{1}{2}$ ins. by 3 ins. by $\frac{3}{8}$ in., the others being 3 ins. by $2\frac{1}{2}$ ins. by $\frac{3}{8}$ in.

To ascertain the approximate effect of a non-axial load, multiply it by 3 when attached to the flange of a rolled joist stanchion, and by $1\frac{1}{2}$ when attached to the web. The virtual load to be carried by this stanchion will then be $10 + 3 \times 10 = 40$ tons. The approximate safe load on a stanchion may be taken for trial as 4 tons per sq. in., then $\frac{40}{4} = 10$ sq. in. as the probable least sectional area, but a stanchion with broad flanges should always be selected. Looking down the table of British Standard Beams, the 8-in. by 6-in. by 35-lb. section with an area of 10·3 sq. ins. appears to be suitable. The radii of gyration of this section are 3·28 and 1·32, and generally it will be necessary to take the least. The safe load per sq. in. on this stanchion by the Rankine-Gordon formula will then be

$$P = \frac{f}{1 + \frac{1}{ac}\left(\frac{l^2}{r^2}\right)} = \frac{6}{1 + \frac{1}{4 \times 9{,}000}\left(\frac{144^2}{1{\cdot}32^2}\right)} = \frac{6}{1{\cdot}33} = 4{\cdot}5 \text{ tons}$$

per sq. in. The maximum stress produced by the loading, axial and eccentric, will be found by the formula $\frac{W}{A} \pm \frac{M}{Z}$ but the minus value may be discarded, as it will not be required in this case. The bending moment M will be half the section depth of stanchion multiplied by the load from the end of girder, and the section modulus Z in the required direction is given as 27·6, therefore $\frac{W}{A} + \frac{M}{Z} = \frac{10 + 10}{10{\cdot}3} + \frac{4 \times 10}{27{\cdot}6} = \frac{20}{10{\cdot}3} + \frac{40}{27{\cdot}6} = 1{\cdot}94$

+ 1·45 = 3·39 tons per sq. in., which is well on the safe side, but the next smaller sections would be the 8 ins. by 5 ins. by 28 lbs., or the 6 ins. by 5 ins. by 25 lbs., both of which have the

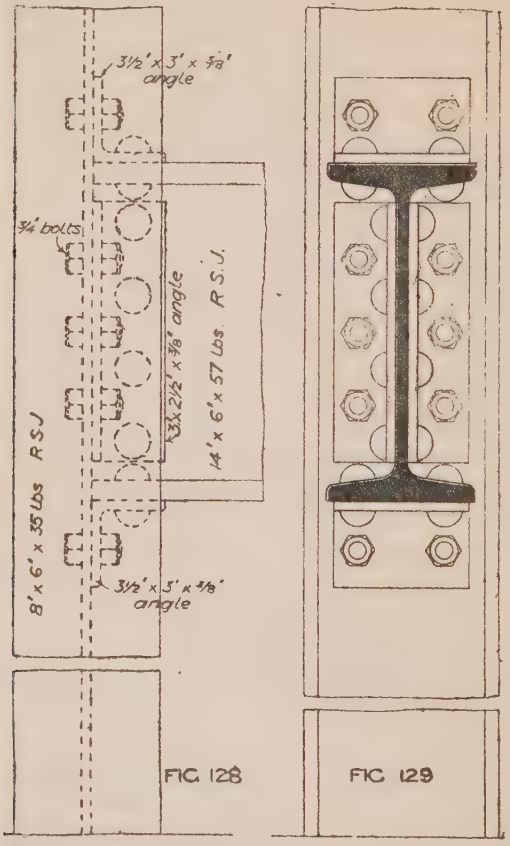


FIG. 128.—Elevation of Stanchion with Girder attached to Web.
FIG. 129.—Section through Girder attached to Web of Stanchion.

defect that they are only 5 ins. wide, while the girder to be connected is 6 ins. wide.

The next case to consider will be the supporting of the girder upon the web of the stanchion as in Figs. 128 and 129. Take the same stanchion 8 ins. by 6 ins. by 35 lbs., then the maximum safe load per sq. in. will be 4·5 tons as before, but the

figures for maximum stress will be altered because the leverage will now only be half the thickness of web, as follows: $\frac{W}{A} + \frac{M}{Z}$ $= \frac{10 + 10}{10{\cdot}3} + \frac{{\cdot}22 \times 10}{5{\cdot}98} = 1{\cdot}94 + {\cdot}368 = 2{\cdot}308$ tons per sq. in., which leaves a still larger margin, but it is seldom possible to provide exactly what is wanted without increasing the cost in other ways.

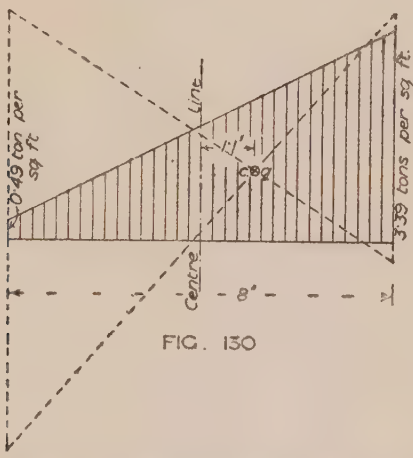


FIG. 130

FIG. 130.—Diagram of Distribution of Stress at foot of Stanchion and Method of finding Centre of Gravity of Diagram.

It will be well to check the maximum allowable stress by the L.C.C. rule. In both cases the ratio $\frac{l}{r} = \frac{144}{1{\cdot}32} = 109$, and the proportionate stress for that value $= 6{\cdot}5 - \frac{1}{40}\left(\frac{l}{r}\right) = 6{\cdot}5 - \frac{1}{40} \times 109 = 6{\cdot}5 - 2{\cdot}725 = 3{\cdot}775$ tons per sq. in., the actual stress in the two cases being 3·39 and 2·308 tons per sq. in. respectively.

When the foundations of a stanchion are concentric with the axis and the stanchion is eccentrically loaded, the foundations will be under more than the calculated stress on the side of the load, and less on the other side. For permanent eccentric loading it is, therefore, desirable to shift the centre of the base plate

and foundation, the distance being found as follows: Taking the case of the girder attached to flange of stanchion $\frac{W}{A} \pm \frac{M}{Z} = 1{\cdot}94 \pm 1{\cdot}45 = +3{\cdot}39$ tons and $+0{\cdot}49$ tons, compression at inner and outer edges. Construct trapezium as Fig. 130 with base equal to depth of stanchion and end heights equal to the stresses just found; this will give a diagram of the distribution of stress at base of stanchion. Find the centre of gravity graphically by adding to each end the height of the other end, and drawing diagonals to

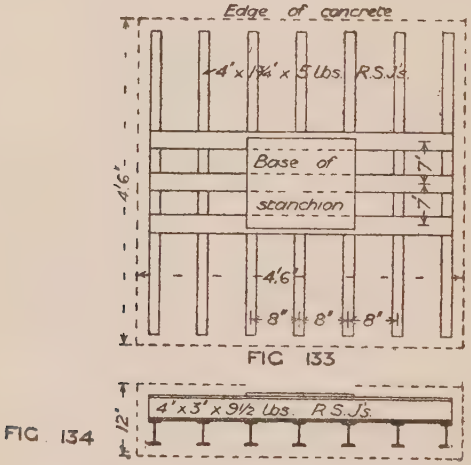


Fig. 133.—Plan of Grillage Foundation to Stanchion.
Fig. 134.—Section of Grillage Foundation to Stanchion.

intersect at the centre of gravity, which will be found to be 1·1 ins. from the centre line of stanchion. If the base plate and foundations are centred on this point they may be designed exactly as for a simple load of the full amount, but otherwise they will require extending. With the girder attached to the web of stanchion the base plate and foundation may be kept central on the axis of the stanchion. The total load is 20 tons, and at 11 tons per ft. super. on the concrete, the steel base plate will require an area of $\frac{20}{11} = 1{\cdot}82$ sq. ft., which will be given by a plate 1 ft. 6 ins. by 1 ft. 3 ins., as shown in elevation Fig. 131 and plan Fig. 132.

When very heavy loads have to be carried upon a poor soil, it is not unusual to construct a grillage foundation to a stanchion. This consists of two, three, or more tiers of rolled joists bedded on and packed between with cement concrete, the projecting portions being calculated as cantilevers. A heavily ribbed cast steel block is often inserted between the stanchion base and top layer of joists to remove any chance of deflection in the base plate. A grillage foundation may be designed for the stanchion calculated above to show the principles, but the steel casting may be omitted as the load is so trifling. Assume the soil to be capable of carrying only 1 ton per ft. super., the load being 20 tons, then the base will have to be 20 sq. ft. area, or if square, say, $\sqrt{20} = 4{\cdot}47$, or 4-ft. 6-in. side as shown in plan Fig. 133 and section Fig. 134. The lower joists will have a projection of $\frac{4{\cdot}5 - 1{\cdot}25}{2} = 1{\cdot}625$ ft., and each will have to resist a total upward pressure of $4{\cdot}5 \times \frac{8}{12} \times 1 = 3$ tons, or $1{\cdot}625 \times \frac{8}{12} \times 1 = 1{\cdot}08$ tons on the cantilever portion. Assuming them to act as cantilevers, the maximum bending moment will be $\frac{WL}{2} = \frac{1{\cdot}08 \times 1{\cdot}625}{2} = 0{\cdot}88$ ton-ft., then the tabular value $= 0{\cdot}88 \times 8 = 7{\cdot}04$, which is given by a (B.S.B. 3) 4-in. by $1\frac{3}{4}$-in. by 5-lb. rolled steel joist. The top joists will have a projection of $\frac{4{\cdot}5 - 1{\cdot}5}{2} = 1{\cdot}5$ ft., and will have to carry two concentrated loads of $\frac{3 \text{ tons}}{3} = 1$ ton each from the lower joists. The maximum bending moment will then be $WL + W'L' = 1 \times 1{\cdot}5 + 1 \times 0{\cdot}7 = 2{\cdot}25$ ton-ft., giving a tabular value of $2{\cdot}25 \times 8 = 18$, which will require a (B.S.B. 4) 4-in. by 3-in. by 9·5-lb. rolled steel joist.

CHAPTER XII

Various Forms of Rivets and Modes of Riveting—Designing Joints for Maximum Efficiency

Rivets and bolts have been called the stitches by which constructional work is put together, rivets being "lock-stitch," only removable by being cut out, and bolts "chain-stitch," easily undone. Riveting may be machine or hand work; in the shop or bridge yard it is now almost entirely machine work, either steam or hydraulic portable riveters being used; out on the job, hand-riveting is customary, except on very large works.

A gang of riveters consists of three men and two boys. Two of the men are riveters and one a holder-up; one of the boys is a forge-boy and the other the carrier, the latter being the younger. The riveters are paid 5*s.* to 6*s.* per day, according to locality; the holder-up, 4*s.* to 4*s.* 6*d.*; boys, 1*s.* to 2*s.* The average wages of the gang may be taken as £1 per day. The average cost per rivet for piecework is as follows: $\frac{1}{2}$ in. = 1*d.*, $\frac{5}{8}$ in. = $1\frac{1}{4}d.$, $\frac{3}{4}$ in. = $1\frac{1}{2}d.$ London prices all through will be somewhat higher. An efficient gang can put in forty $\frac{3}{4}$-in. rivets per hour if the platers have the work ready prepared for them. In outdoor work the rivets are heated by putting them through holes in a plate placed over the hollow of the fire in a small rivet forge, so that the shanks may be made red hot while the heads are kept cool. Care has to be taken that the rivets are not over-heated or burnt, the forge-boy being responsible for this. The platers connect by platers' bolts the parts which are to be riveted together, drawing them as close together as possible. In the case of an open joint the riveters flog the plates—that is, hammer them close to prevent a collar being formed on the rivet between the plates. The forge-boy picks the rivets one at the time out from the perforated plate, as the riveters are ready, throws it on the ground to knock the clinker off, and the carrier

runs with it to the holder-up and dips the point in water before throwing it down to him. The holder-up then picks up the rivet, pushes it through the hole, and holds the dolly against it; the riveters then strike on the point, and the first effect is to swell out the middle of the rivet, where it is hottest, to fill the hole; by that time the heat has crept up to the point, and the force of the hammer knocks it down into a roughly-rounded shape. One riveter then holds the snap, or cup tool, on the roughly-formed head while the other strikes with the hammer to give it a perfect shape. In girder work riveting should commence at the outer ends and work towards the middle.

There is no difference in the general appearance between machine- and hand-made rivets, except that in machine rivets there is more often a frill left round the new head. On cutting open and examining the joint, a machine rivet shows the hole to be much better filled, and, testing the two modes of riveting, it is found that machine riveting is stronger, because of the greater sectional area of the rivet and the better grip upon the plates. The only disadvantage in machine riveting is a greater tendency to burst the holes from the fluid pressure generated at the centre of the rivet, the remedy for this being not to make the rivets quite so hot; and it is essential to keep the cup tool of the machine upon the rivet for a few seconds after the rivet is made, in order that a tight joint may be secured.

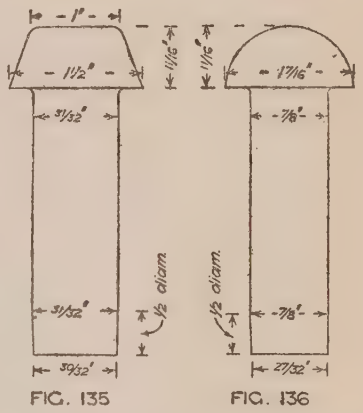


FIG. 135. FIG. 136.

FIG. 135.—Standard proportion of Pan-head Rivet for 1-in. hole.

FIG. 136.—Standard proportion of ⅞-in. Snap-head Rivet.

There is some confusion as to the proper size of rivets; for instance, a ¾-in. rivet may be ¾ in. diameter, when the hole must be larger, so as to allow for the expansion of the rivet when red-hot, and the finished rivet will be more than ¾ in. diameter and, therefore, stronger than the calculated value, but the sectional

area of the plates will be reduced by the difference in diameter. On the other hand, if the hole is $\frac{3}{4}$ in. diameter the rivet must be less, but the finished diameter of rivet will agree with the calculations. There is no rule as to which method shall be adopted, but it would be convenient for the nominal diameter always to represent that of the hole. The difference between the two diameters varies from $\frac{1}{32}$ in. to $\frac{1}{16}$ in. Mr. Henry Fidler, of the Admiralty Works Department, gives the dimensions of two standard rivets as shown. Fig. 135 is called "pan head,"

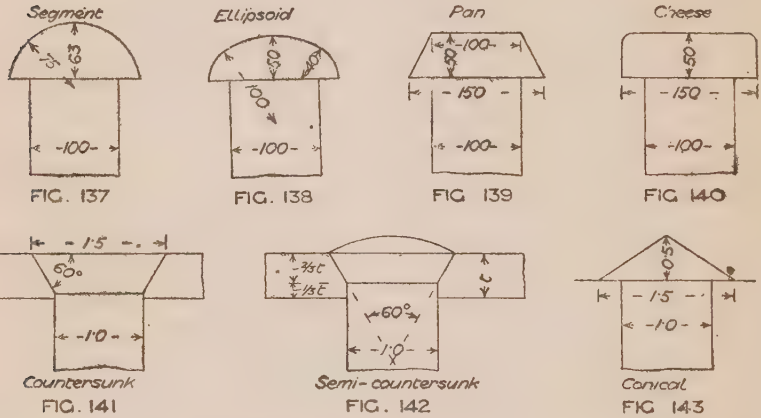


FIG. 137.—Rivet with Segmental Head. FIG. 138.—Rivet with Elliptical Head.
FIG. 139.—Rivet with Pan Head. FIG. 140.—Rivet with Cheese Head.
FIG. 141. — Countersunk Rivet. FIG. 142. — Semi-countersunk Rivet.
FIG. 143.—Rivet with Conical Head.

and Fig. 136 "snap" or "cup-head"; the small curve under the head is important, and requires the arris taken off the hole. It is a matter of taste which sort of head is used, and the shape is quite independent of the question of diameter. It will be observed that for a length equal to half the diameter the point is tapered so that it will enter the hole easily. Pan-head rivets are often used in boiler and tank work, but in girder work snap heads are usual. The standard proportions for rivet heads given in Molesworth's Pocket Book are shown in Figs. 137 to 140. The point of a rivet finished by hammering only is either countersunk flush with plate, as Fig. 141, or semi-countersunk, as Fig. 142, or

conical, as Fig. 143. The latter is a bad shape for resisting tension, and not often seen at the present day. The usual form is the snap head produced by a cupped tool called a "snap"; and Fig. 138 is best, because it has full strength against tension and the minimum projection.

Many of the published tables are for iron rivets in iron plates,

FIG. 144.—Chain Riveting.

FIG. 145.—Staggered or Zig-zag Riveting.

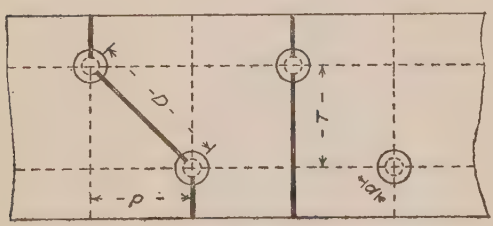


FIG. 146.—Minimum Pitch for Staggered Riveting.

but steel rivets in steel plates are now customary. The working strength of steel rivets has already been given.

In boiler, tank, and girder work, when more than one row of rivets is used through the same piece, there are two modes of placing them: Fig. 144, known as "chain" riveting, and Fig. 145, known as "staggered" or "zig-zag." Generally, the hole for a rivet must be at least its own diameter away from the edge, and

the pitch or distance from centre to centre about $2\frac{1}{2}$ or 3 diameters in boiler and tank work, and four diameters in constructional work. For staggered riveting the minimum longitudinal pitch p, without weakening the cross-section, occurs when the diagonal pitch D equals the sum of the diameter of rivet d and transverse pitch T, then $D = d + T$, as shown in Fig. 146; also $T = D - d$, and $p = \sqrt{D^2 - T^2}$. For example, let $T = 2{\cdot}5$ ins., $d =$ 1 in., then $D = 1 + 2{\cdot}5 = 3{\cdot}5$ ins., $p = \sqrt{3{\cdot}5^2 - 2{\cdot}5^2} = \sqrt{12{\cdot}25 - 6{\cdot}25} = \sqrt{6} = 2{\cdot}45$ ins. The amount to tear through will then be the same, whether it occurs straight across or partly diagonally, as shown by the thick lines.

In boiler and tank work there are two kinds of joints, "lap" and "butt"; these, again, may be "single-" or "double-" riveted—that is, with one or two rows of rivets in each, and chain or zig-zag; but in the double-riveted butt joints there will be two rows on each side of the joint, and the butt joint may be covered by single or double butt strips. These are shown in Figs. 147 to 152.

These joints should be designed for equal resistance to failure in each part. An example may be taken of a single-riveted lap joint. Let

d = diameter of rivets in inches.
p = pitch of rivets in inches.
t = thickness of plate in inches.
T = tensile strength of plate in tons per sq. inch.
S = shearing resistance of rivet per sq. inch.
B = resistance to crushing per sq. inch.
R = resistance of strip of joint of width p in tons.
π = Ratio of circumference to diameter = 3·1416 or $\frac{22}{7}$.
e = efficiency per cent. of solid plate.

Resistance to tension $R = (p - d)\,tT$ . . . (1)

„ „ shearing $R = \frac{\pi}{4} d^2 S$ . . . . . (2)

„ „ crushing $R = dtB$ . . . . . (3)

Equating (2) and (3)

$$\frac{\pi}{4} d^2 S = dtB \therefore d = \frac{4\,tB}{\pi S} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

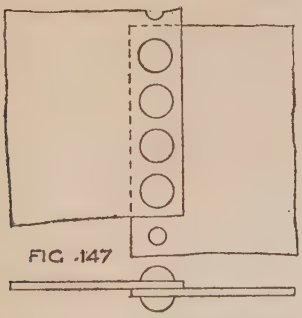


FIG. 147.—Single-Riveted Lap Joint.

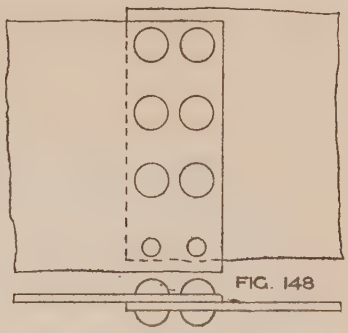


FIG. 148.—Double-Riveted Lap Joint.

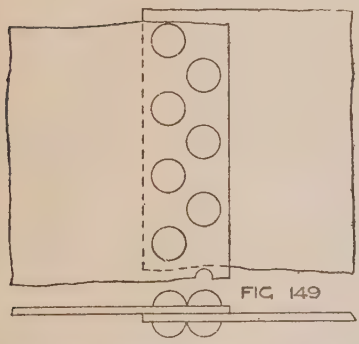


FIG. 149.—Zig-zag-Riveted Lap Joint.

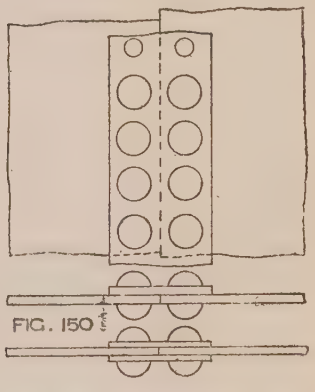


FIG. 150.—Single-Riveted Butt Joint.

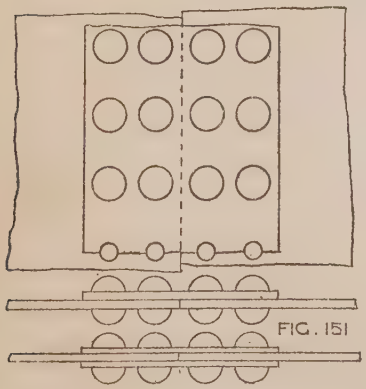


FIG. 151.—Double-Riveted Butt Joint.

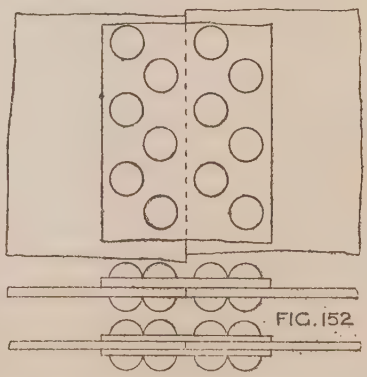


FIG. 152.—Zig-zag-Riveted Butt Joint.

Equating (1) and (2)

$$(p - d)\, tT = \pi\, d^2 S \therefore p = \frac{\pi d^2 S}{4\, tT} + d \quad . \quad . \quad . \quad (5)$$

Substituting in (5) value for d found in (4)

$$p = \frac{4\, tB\, (B + T)}{\pi ST} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$\text{Efficiency } e = 100\,\frac{p - d}{p} \text{ per cent.} \quad . \quad . \quad . \quad . \quad (7)$$

For example: Let $t = \frac{1}{2}$ in., $T = 6\frac{1}{2}$ tons, $S = 5$ tons, and $B = 10$ tons.

Then by (4) $d = \dfrac{4 \times \frac{1}{2} \times 10}{\frac{22}{7} \times 5} = 1{\cdot}27$ ins.

and by (6) $p = \dfrac{4 \times \frac{1}{2} \times 10\,(10 + 6\frac{1}{2})}{\frac{22}{7} \times 5 \times 6\frac{1}{2}} = 3{\cdot}23$ ins.

and by (7) $e = 100 \times \dfrac{3{\cdot}23 - 1{\cdot}27}{3{\cdot}23} = 60{\cdot}7$ per cent.

which is the greatest possible efficiency for a single-riveted lap joint with steel plates and rivets at working intensities of stress.

Another form of riveted joint occurs when the ends of a bar or strap have to be joined, as in a hoop round a dome, or any flat tension members in a roof, bridge, or other structure. To illustrate the mode of designing such a joint an investigation of the strongest form may be made for 9-in. by $\frac{3}{8}$-in. steel bars with $\frac{3}{4}$-in. steel rivets under safe working stresses. The calculations will proceed in stages as follows:—

Safe tensile stress on solid bar

$= 9 \times \frac{3}{8} \times 6\frac{1}{2} =$ . . . . 21·9375 tons.

Safe shearing stress on one $\frac{3}{4}$-in. rivet in drilled hole

$= {\cdot}4417 \times 5 =$ . . . . . 2·2085 tons.

Safe bearing pressure on one $\frac{3}{4}$-in. rivet in drilled hole in $\frac{3}{8}$-in. plate

$= \frac{3}{4} \times \frac{3}{8} \times 10 =$ . . . . 2·81 tons.

Safe tensile stress on plate through one rivet hole

$= (9 - \frac{3}{4}) \times \frac{3}{8} \times 6\frac{1}{2} =$ . . . 20·11 tons.

Safe tensile stress on plate through two rivet holes when one rivet beyond is under full bearing pressure

$= (9 - 1\frac{1}{2}) \times \frac{3}{8} \times 6\frac{1}{2} + 2{\cdot}81 =$. 21·09 tons.

Safe tensile stress on plate through three rivet holes when three rivets beyond are under full bearing pressure

$= (9 - 2\frac{1}{4}) \times \frac{3}{8} \times 6\frac{1}{2} + 3\ (2{\cdot}81) = 16{\cdot}45 + 8{\cdot}43 =$ 24·88 tons.

Safe tensile stress on plate through three rivet holes when three rivets beyond are under full shear allowance

$= 16{\cdot}45 + 3\ (2{\cdot}2085) =$ . . . 23·08 tons.

The lowest value for strength of joint as found above is 20·11 tons. Number of rivets required $= \frac{20{\cdot}11}{2{\cdot}2085} = 9{\cdot}1$ rivets. This is so nearly nine rivets that it will probably be quite safe to take that number, and the joint will then appear as Fig. 152A.

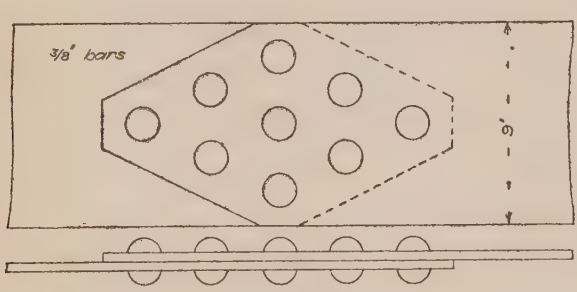


FIG. 152A.—Riveted Joint for two Bars.

The strength of the finished joint may now be tested as follows:—

Safe total shear strength of rivets

$= 9 \times 2{\cdot}2085 =$ . . . . 19·8765 tons.

Safe total bearing pressure of rivets

$= 9 \times 2{\cdot}81$ . . . . . 25·29 tons.

Safe total tensile stress on plate through first rivet hole

$=$ . . . . . 20·11 tons.

So that the full strength of the joint may be taken as 20 tons, and the percentage of efficiency as $100 \times \frac{\text{least resistance}}{\text{strength solid plate}}$

$= \frac{100 \times 19{\cdot}8765}{21{\cdot}9375} = 90{\cdot}6$ per cent.

Before closing this chapter it may be well to give a brief specification for structural steel, but reference for further

information should be made to the British Standard Specifications, Nos. 13, 14, and 15, for structural steel for various purposes.

"All structural steel must be manufactured by the open-hearth acid process, and must not show on analysis more than ·06 per cent. of sulphur or phosphorus. Bessemer steel will not be accepted. The ultimate tensile strength must be from 28 to 32 tons per sq. in., and elongation not less than 20 per cent. in 8 ins. Samples must bend without cracking through an angle of 180 degs. to an inside radius of $1\frac{1}{2}$ times the thickness of test piece, either hot or cold, or after being heated to a cherry-red and cooled in water at about 82° F. For rivets the tensile stress must be from 26 to 30 tons and the elongation in an 8-in. length 25 per cent." Care must be taken in considering elongations to note whether they are upon 10-in., 8-in., $6\frac{1}{4}$-in., or 2-in. lengths, as all of these are found in use, and the elongation will be proportionately greater on the shorter lengths.

CHAPTER XIII

Lattice Girders—Various Types of Girders with Stress Diagrams

Lattice Girders are much in vogue for carrying the roof trusses of railway stations, factories, warehouses, and other places where support over long spans is required with the minimum of obstruction to the light. Also for foot-bridges, because their depth forms a good protection for the sides, taking the place of a parapet wall; on a larger scale they are used for road and railway bridges, and may involve work of the heaviest class. There are various kinds of lattice girders; the simplest is the Warren girder, having single triangulations as in Fig. 153, the stress diagram for which under uniform loading is given in Fig. 154. The thick lines in the frame diagram indicate compression, the thin lines tension, and the dotted lines no stress. This form is frequently used for purlins of long span, the lattice bars being placed at 60 degs., and generally forming eight or ten bays; fewer bays are shown here in order to reduce the complication of the diagrams. The stress diagram takes account only of the loads coming on to the points of support of the flange; when the load is distributed uniformly along the flange there will be a bending moment to allow for, in addition to the direct stress, and the ultimate stress will be calculated by the formula $\frac{W}{A} \pm \frac{M}{Z}$.

The next form, Fig. 155, is, perhaps, the type most often seen, but with a greater number of bays; it consists of a double series of triangulations connected at the crossing of the diagonal bars, which are at an angle of 45 degrees. The stress diagram is shown in Fig. 156. Point 8 in the stress diagram is found by observing that the load passing through bar 1—8 is the whole of load 1—2, and half of load 3—4. To tell whether a bar is in tension or compression, take the junction with a force of known direction,

as force 2—3 in Fig. 155, then look at the stress diagram and follow clockwise with the bars meeting at that junction, and notice the direction in which the line reads with regard to the junction point. Thus 2—3 downwards, 3—12 towards the joint (compression), 12—11 same point (no stress), 11—9 towards the joint (compression), 9—2 towards the joint (compression). Or

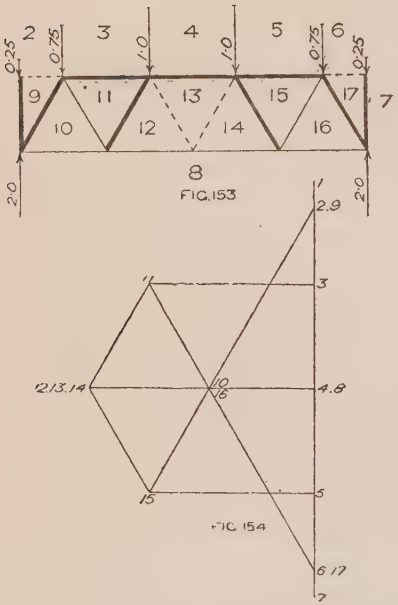


FIG. 153.—Frame diagram of Warren Girder.

FIG. 154.—Stress diagram for Fig. 153.

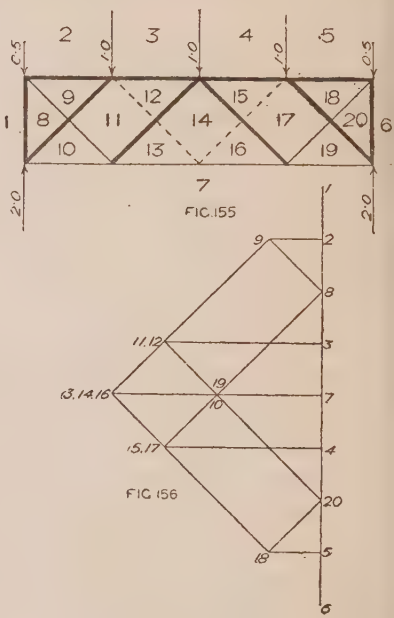


FIG. 155.—Frame diagram of Lattice Girder with simple Cross-bracing.

FIG. 156.—Stress diagram for Fig. 155.

take the top left-hand corner of Fig. 155, 1—2 downwards, 2—9 towards the joint (compression), 9—8 away from the joint (tension), 8—1 towards the joint (compression). Inspecting the direction in the stress diagram in this way, and marking the compression members of the frame diagram is better than marking arrow heads upon the stress diagram and bars, as is sometimes recommended. There is not a very great difference in the appearance of the next type, Fig. 157, which has a vertical bar at each apex of the triangulation, but these are "redundant

members" as regards the simple distribution of stress, and the actual stresses in the various members will depend very largely upon the workmanship. In one of the American treatises on graphic statics, twenty pages are occupied in elucidating the stresses by means of the calculus, but a simple practical method is generally adopted of assuming that half the apex load passes

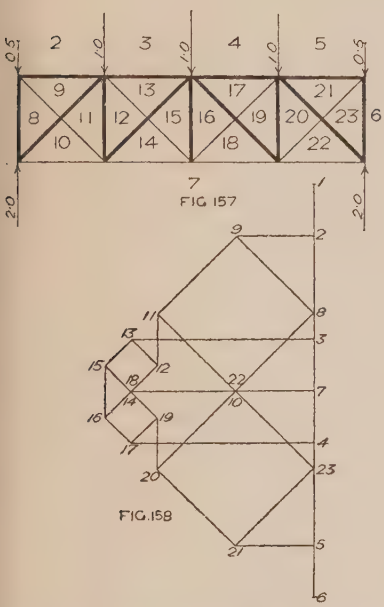


FIG. 157.—Frame diagram of Lattice Girder with Cross-bracing and Verticals.
FIG. 158.—Stress diagram for Fig. 157.

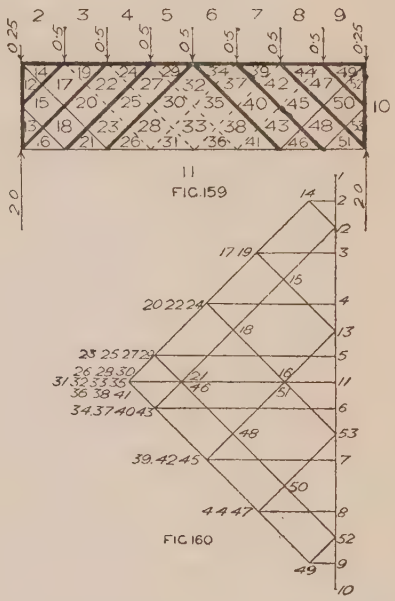
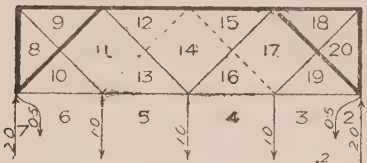


FIG. 159.—Frame diagram of Trellis Girder.
FIG. 160.—Stress diagram for Fig. 159.

through each vertical, making the stress diagram as Fig. 158. Point 8 in the stress diagram will be found by observing that the whole of load 1—2, $\frac{3}{4}$ of $\frac{1}{2}$ load 2—3, $\frac{1}{2}$ of $\frac{1}{2}$ load 3—4, and $\frac{1}{4}$ of $\frac{1}{2}$ load 4—5 passes down 1—8. The reason for $\frac{3}{4}$ of $\frac{1}{2}$ load 2—3 is due to its position; calculating by leverage $\frac{3}{4}$ would be carried by the left abutment, and $\frac{1}{4}$ by the right abutment. Load 4—5 is under exactly reverse conditions. The stress in 6—23 will be equal to that in 1—8, as the loading is symmetrical. This type of girder is sometimes preferred, because it is thought that the

vertical bars produce a better distribution of the stress, and add to the general stiffness. A very limited number of bays has been taken in these examples, in order that space may not be unnecessarily occupied with diagrams, but it is essential that the student should work out other examples with varying numbers of bays, as many new points will arise. In practice it is usual to terminate lattice girders with a plate web, or "pillar," over



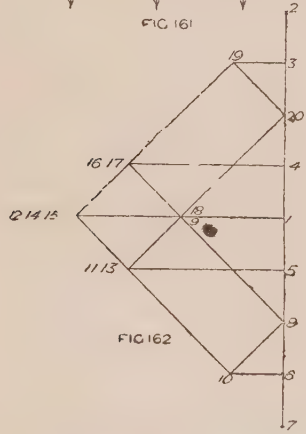
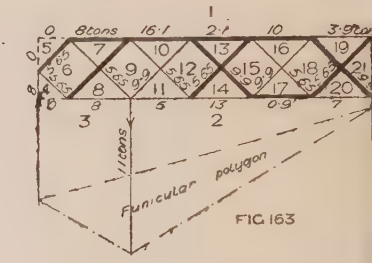


FIG. 161.—Frame diagram of Lattice Girder with Cross-bracing and Load on Bottom Flange.

FIG. 162.—Stress diagram for Fig. 161.



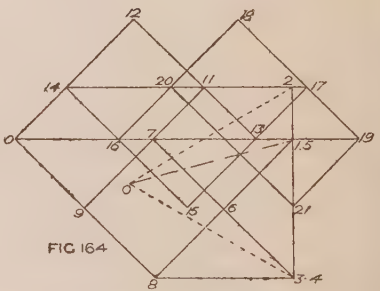


FIG. 163.—Frame diagram of Lattice Girde irregularly loaded.

FIG. 164.—Stress diagram for Fig. 163.

the supports, so that the diagonal bracing is usually co-terminous with the clear span. The remaining type consists of a multiple series of lattices, forming a trellis girder as Fig. 159, the stress diagram being as Fig. 160, but the number of lattice bars may be increased to any extent, with a corresponding reduction of the stress in the individual bars. Point 12 will be found by taking the load passing down 1—12, as the whole of 1—2 and half of 5—6, and point 13 by taking the whole of 1—12, $\frac{7}{8}$ of 2—3, $\frac{5}{8}$ of 4—5, $\frac{3}{8}$ of 6—7, and $\frac{1}{8}$ of 8—9.

If the load be applied to the bottom flange of a lattice girder, the procedure will be very similar, the loads being shown as external forces at the bottom of the girder. As an example, Fig. 153 may be repeated with the load on the bottom flange as Fig. 161. In the stress diagram Fig. 162, bar 1—8 has $\frac{3}{4}$ of load 5—6, and $\frac{1}{4}$ of 3—4, and on completion of the figure it will be found that more bars are in tension, and that, therefore, on the whole, the girder would be more economical. The question of cost being often paramount, various types of braced structures for a given case may be compared by a method suggested by Mr. Thomas Gillott (Trans. Soc. Eng., 1881). He compares the value of different types by taking the products of the lengths of the various members by the stress upon them, dividing the sum of those in tension by 4 tons per sq. in., and the sum of those in compression by $1\frac{1}{4}$ tons per sq. in. The results added together will give the comparative weights for each type, and the cost will vary nearly as the weight. These figures were given for wrought iron, and represent approximately the mean intensity of stress on the various members. There will be no appreciable error introduced in using the same figures for steel, as the results give comparative and not actual weights.

Some very curious diagrams are occasionally found when the loading is irregular. For example, Fig. 163 was given in *Engineering*, January 4, 1895, as the frame diagram of a lattice girder carrying a load of 11 tons at one apex on bottom flange. The proper stress diagram for this is shown in Fig. 164. The dotted lines in both figures show the method of finding the reactions by means of a link or funicular polygon, as follows:— Set down the load line 2—3, and select any pole 0, Fig. 164, and draw vectors to points 2 and 3. Then on Fig. 163 across spaces 2 and 3 draw lines parallel with the vectors 0—2 and 0—3, and join the extremities. Parallel with this last line draw a vector from pole 0 to give point 1, thus fixing the amount of each reaction 1—2 and 3—1.

A complete design will be given for a lattice girder in the next chapter, with the method of determining the formation of the joints.

CHAPTER XIV

Designing Lattice Girder for Supporting Roof Trusses—Details of Joints and How Arrived At

An example will now be taken of a mild steel lattice girder, as one of a series, for supporting the roof over a factory, say 80 ft. or 100 ft. long by 40 ft. wide. It is assumed that the trusses are 10 ft. from centre to centre and 20 ft. span, so that the girders which carry the trusses will be 20 ft. from centre to centre and 40 ft. span. The horizontal area supported by each truss, whether "north light" or otherwise, will be 20 ft. by 10 ft., or 200 sq. ft., which at an approximate total load of $\frac{1}{2}$ cwt. per foot super will bring a series of loads of $\frac{200 \times \frac{1}{2}}{20} = 5$ tons each on the girder. The number of bays in the girder should be so arranged that each truss comes upon an apex; there will thus be eight or twelve bays. As the angle of the lattice bars should be about 45 degs., eight bays would make the girder 5 ft. deep and twelve bays 3 ft. 4 ins. deep. The latter size would be preferable, and may be adopted. It may also be assumed that two lines of shafting run down the factory and have to be carried by hangers bolted to the bottom flange of girder. These may be placed under the second apex from each end, say 6 ft. 8 ins. from wall. The load due to these shafts will consist of their weight and the hangers and plummer blocks, and of the pull of the belts. It is somewhat difficult to estimate the amount, but the engineers supplying the machinery will generally give an estimate, which should always be an ample one; it may be taken in this case at $2\frac{1}{2}$ tons for each line of shafting. If the hangers are bolted between two apices the load must be divided between them, and the bottom flange stiffened in that bay to withstand the cross-strain. The frame diagram will be as shown in Fig. 165, and the stress diagram as Fig. 166. The only difficulty arising is to know the

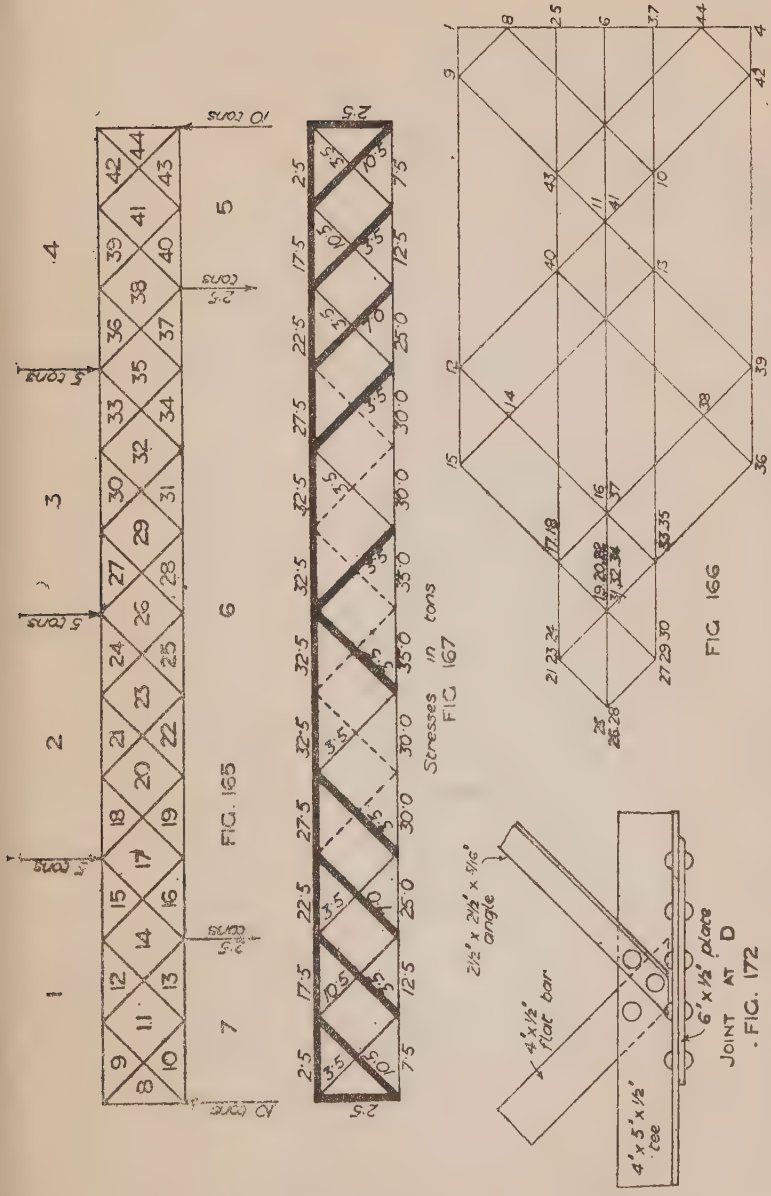


FIG. 165.—Frame diagram of Lattice Girder with Loads from Roof Trusses and Shafting. FIG. 166.—Stress diagram for Fig. 165. FIG. 167.—Frame diagram with Stresses marked on. FIG. 172.—Detail of Joint at D.

stress in bar 1-8; but it will be found as described in the last chapter. It arises from the pull of bar 8-9, which transmits $\frac{1}{2}$ of 2-3. Fig. 167 shows the compression members by thick lines and the tension members by thin lines, the amount of stress being marked upon each. Only one-half need be marked, as both sides of the centre will be the same. The bars will be connected where they cross, so as to reduce the unsupported length of the compression members. The calculations will be as follows: For the first three diagonal compression members from either support, with a maximum stress of 10·5 tons and unsupported length of, say, 2 ft. 4 ins. = 28 ins., try a 4-in. by 3-in. by $\frac{3}{8}$-in. angle iron, of which the least width will be 2·4 ins.; then, by Gordon's formula, assuming the struts to be equivalent to "fixed one end and rounded the other,"

$$p = \frac{f}{1 + \frac{m}{nq}\left(\frac{l}{d}\right)^2} = \frac{26}{1 + \frac{2{\cdot}5}{\frac{1}{2} \times 3{,}750}\left(\frac{28}{2{\cdot}4}\right)^2} = \frac{26}{1{\cdot}18} = 22 \text{ tons}$$

per sq. in. ultimate load. The factor of safety $= 4 + {\cdot}05\left(\frac{l}{d}\right)$ $= 4 + {\cdot}05\left(\frac{28}{2{\cdot}4}\right) = 4{\cdot}58$, then $\frac{22{\cdot}0}{4{\cdot}58} = 4{\cdot}8$ tons per sq. in. safe load. The area = 2·48 sq. ins., therefore the total safe load $= 4{\cdot}8 \times 2{\cdot}48 = 11{\cdot}9$ tons, so that this section will be sufficient. For the next three diagonal compression members on either side, with a maximum compression of 3·5 tons, and unsupported length of 28 ins., try a $2\frac{1}{2}$-in. by $2\frac{1}{2}$-in. by $\frac{5}{16}$-in. angle iron. Then

$$p = \frac{f}{1 + \frac{m}{nq}\left(\frac{l}{d}\right)^2} = \frac{26}{1 + \frac{2{\cdot}5}{\frac{1}{2} \times 3{,}750}\left(\frac{28}{1{\cdot}8}\right)^2} = \frac{26}{1{\cdot}32} = 19{\cdot}7 \text{ tons}$$

per sq. in. ultimate load. The factor of safety $= 4 + {\cdot}05\left(\frac{l}{d}\right)$ $= 4 + {\cdot}05\left(\frac{28}{1{\cdot}8}\right) = 4{\cdot}77$, then $\frac{19{\cdot}7}{4{\cdot}77} = 4{\cdot}13$ tons per sq. in. safe load, and as the area = 1·47 sq. in. the total safe load = $4{\cdot}13 \times 1{\cdot}47 = 6$ tons. This is nearly double the value of the maximum stress in the bars, but the section should not be smaller owing to the size of the rivet holes required through them, and the necessity for keeping up a certain size to resist corrosion, which is more

disastrous on a small section. For the first three diagonal tension members from either support with a maximum stress of 10·5 tons, try a 4-in. by $\frac{1}{2}$-in. flat bar. Then the sectional area deducting one $\frac{3}{4}$-in. rivet hole = 1·625 sq. ins., and at $6\frac{1}{2}$ tons per sq. in. the safe load = 1·625 × $6\frac{1}{2}$ = 10·56 tons, so that this size will be just enough. For the next three diagonal tension members on either side, with a maximum stress of 3·5 tons, try a $2\frac{1}{2}$-in. by $\frac{5}{16}$-in. flat bar. The sectional area less one rivet hole = 0·547 sq. ins. and the safe load = 0·547 × $6\frac{1}{2}$ = 3·55 tons, so that this section will be sufficient.

The lower flange has a maximum tension of 35 tons, and at $6\frac{1}{2}$ tons per sq. in. will require to have a net area of $\frac{35}{6{\cdot}5}$ = 5·4 sq. ins. The largest standard tee section is 4 ins. by 5 ins. by $\frac{1}{2}$ in., which has a sectional area of 4·25 sq. ins., then, deducting two $\frac{3}{4}$-in. rivet holes from the flange and one from the web, the net sectional area left = 4·25 − (3 × $\frac{3}{4}$ × $\frac{1}{2}$) = 3·125 sq. ins. This is not sufficient, and a 6-in. by $\frac{1}{2}$-in. plate must be added, the area less two rivet holes being 6 × $\frac{1}{2}$ − (2 × $\frac{3}{4}$ × $\frac{1}{2}$) = 2·25 sq. ins., making a total of 3·125 + 2·25 = 5·375 sq. ins., which is practically the figure required. The tee section alone less one rivet hole will be capable of resisting a tensile stress of 3·875 × 6·5 = 25·2 tons, so that the extra plate need not be continued over the whole span, but may be stopped three bays from each end, as the stresses in these bays are less than 25·2 tons; it must, however, run beyond the joint, in order to secure its efficiency for the previous bay. The hangers for shafting must be bolted up through rivet holes when they occur in a convenient place, otherwise they must be bolted by clips so as not to weaken the flange.

The same section with plate may be tried for the top flange, which has a maximum stress of 32·5 tons and an unsupported length of, say, 10 ft.—that is, the distance between the roof principals supported by the girder. Then

$$p = \frac{f}{1 + \frac{m}{nq}\left(\frac{l}{d}\right)^2} = \frac{26}{1 + \frac{2{\cdot}5}{\frac{1}{2} \times 3{,}750}\left(\frac{10 \times 12}{6}\right)^2} = \frac{26}{1{\cdot}53} = 17$$

tons per sq. in. ultimate load, the factor of safety = 4 + ·05

$\left(\frac{l}{d}\right) = 4 + \cdot 05\left(\frac{120}{6}\right) = 5$, therefore the safe load per sq. in. $= \frac{17}{5} = 3{\cdot}4$ tons. The top flange is in compression, and the rivet holes need not be deducted, so that the area $= 4{\cdot}25 + 3 = 7{\cdot}25$ sq. ins. and the total safe load $= 7{\cdot}25 \times 3{\cdot}4 = 24{\cdot}65$ tons. This is not enough to take the maximum stress, and the addition of another 6-in. by $\frac{1}{2}$-in. plate may be tried. The length and least diameter are not altered, so that the safe stress per sq. in. will be 3·4 tons, as before, the area $= 4{\cdot}25 + 3 + 3 = 10{\cdot}25$ sq. ins., and the total safe load $10{\cdot}25 \times 3{\cdot}4 = 34{\cdot}85$ tons, so that this section will be sufficient. It will be seen from the stresses marked on Fig. 167 that the inner plate may obviously stop short one bay from each support, and the outer plate may stop three bays from each support.

The end vertical members have a stress of 2·5 tons, and unsupported length of 3 ft. 4 ins. = 40 ins., and a $2\frac{1}{2}$-in. by $2\frac{1}{2}$-in. by $\frac{5}{16}$-in. angle section may be tried. Then

$$p = \frac{f}{1 + \frac{m}{nq}\left(\frac{l}{d}\right)^2} = \frac{26}{1 + \frac{2{\cdot}5}{\frac{1}{2} \times 3{,}750}\left(\frac{40}{1{\cdot}8}\right)^2} = \frac{26}{1{\cdot}65} = 15{\cdot}76 \text{ tons}$$

per sq. in. ultimate load, factor of safety $= 4 + \cdot 05\left(\frac{40}{1{\cdot}8}\right) = 5{\cdot}1$, therefore safe load $= \frac{15{\cdot}76}{5{\cdot}1} = 3{\cdot}09$ tons per sq. in. Area of section $= 1{\cdot}47$ sq. in., so that the total safe load $= 1{\cdot}47 \times 3{\cdot}09 = 4{\cdot}54$ tons, which is more than enough, but, for the reasons previously stated, this should be the smallest size adopted, and they would generally be put in duplicate as shown.

The next step in the calculations will be the riveting; area of $\frac{3}{4}$-in. rivet $= 0{\cdot}4417$ sq. ins., at 5 tons per sq. in., single shear, each $\frac{3}{4}$-in. rivet will take $5 \times 0{\cdot}4417 = 2{\cdot}2$ tons. Therefore members with a stress of 10·5 tons will require $\frac{10{\cdot}5}{2{\cdot}2} = 5$ rivets, members with a stress of 7 tons $\frac{7{\cdot}0}{2{\cdot}2} =$ say, 4 rivets, and members with a stress of 3·5 tons, $\frac{3{\cdot}5}{2{\cdot}2} =$ say, 2 rivets. It will be seen that in order to keep the joints as compact as possible, the least

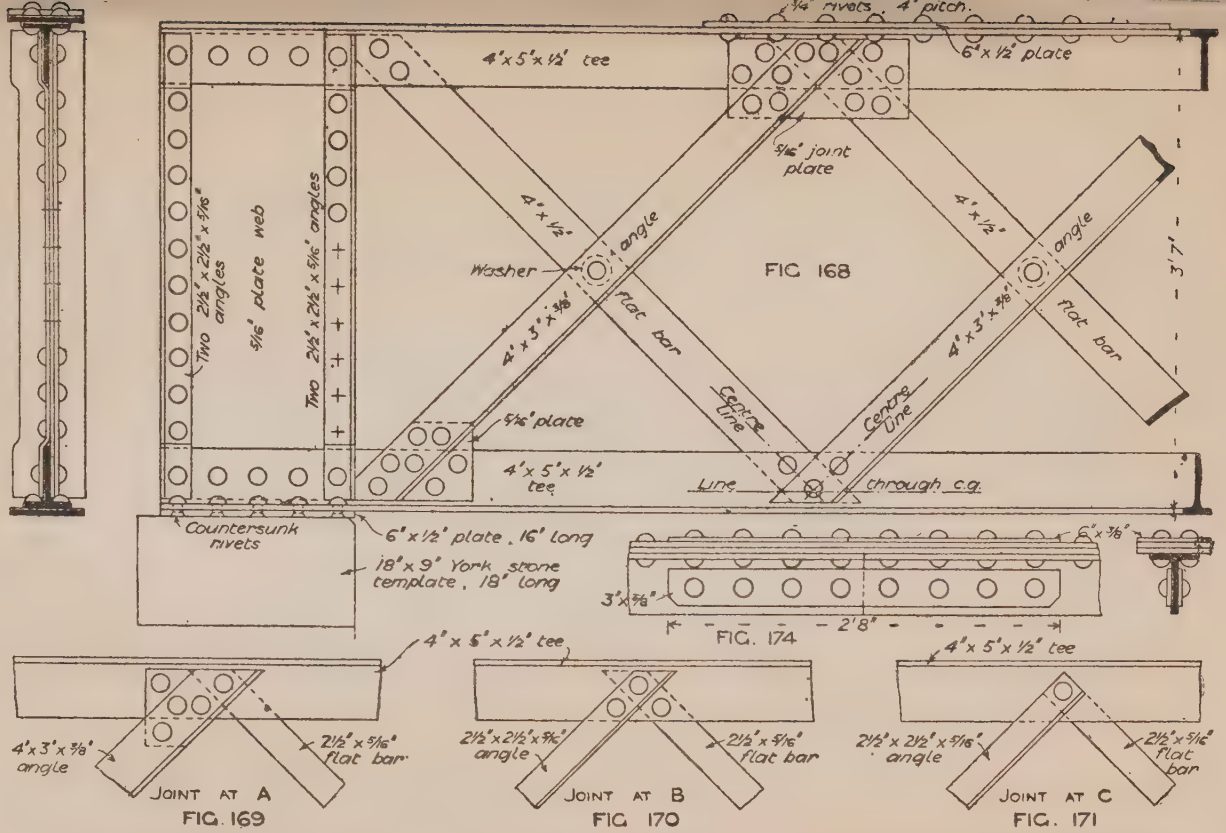


FIG. 168.—Elevation of end of Girder. FIG. 169.—Detail of Joint at A, Fig. 173. FIG. 170.—Detail of Joint at B.
FIG. 171.—Detail of Joint at C. FIG. 174.—Elevation and Section of Joint in Flange.

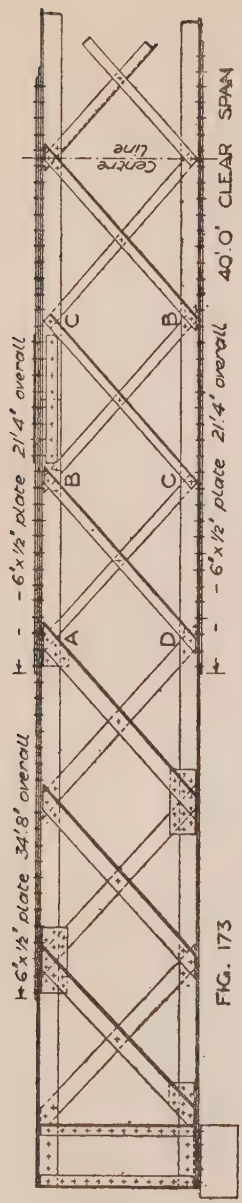


FIG. 173.—Half elevation of Girder.

allowable pitch of rivets should be adopted where convenient. Another limiting rule is that all rivet holes should be at least one diameter away from the edge of section, so that with $\frac{3}{4}$-in. rivets the centre of rivet must not be less than $\frac{3}{4} + \frac{3}{8} = 1\frac{1}{8}$ ins. away from edge of section, and where more than one line of rivets is required in tension members, the diagonal pitch must be proportioned as previously explained. Theoretically, the centre of gravity lines of the various members meeting in a joint should intersect in one point; but this is sometimes a little inconvenient, and no great harm will ensue in the production of secondary stresses if the centre line of a compression member is adopted instead of the centre of gravity line. In the case shown in Fig. 168 the centre lines of the tension and compression members intersect on the centre of gravity line of the flanges.

The frame diagram represents the skeleton of the structure, and in clothing the skeleton the alteration in dimensions must be noticed. The intersection of last lattice bar at each end will be about $1\frac{1}{2}$ ins. from the face of wall, then 40 ft. minus twice $1\frac{1}{2}$ ins. = 39 ft. 9 ins., making the actual width of each bay 3 ft. $3\frac{3}{4}$ ins., which will also be the depth between the centre of gravity lines of the flanges. The distance to bottom of flange being 1·6 ins., the total depth of girder on the outside of the tees will be 3 ft. $3\frac{3}{4}$ ins. + 3·2 ins., say $3\frac{1}{4}$, = 3 ft. 7 ins.

The load at each support will be 10 tons, and allowing 15 tons per sq. ft. on the York stone template, the bearing area required will be $\frac{10 \text{ tons}}{15 \text{ tons per sq. ft.}} = 0{\cdot}66$ sq. ft. As the bearing plate is 6 ins. $= 0{\cdot}5$ ft. wide, the length required $= \frac{0{\cdot}66}{0{\cdot}5} = 1{\cdot}33$ ft. $= 1$ ft. 4 ins., and the stone may be 18 ins. square by 9 ins. deep.

The arrangement of the joints must now be sketched out, and the elevation of the end of girder drawn to a fairly large scale, as in Fig. 168. The remaining joints, where any difference occurs, will be drawn out as Figs. 169 to 172. The whole girder will then appear as shown in half elevation Fig. 173. It must be noted that it is not usual to proportion the lattice bars exactly to the stress; there should be as much repetition as possible in the sizes, to avoid expense, and in this case it will be seen that only one change is made. These girders are generally finished off with a web plate at the ends as shown, which is sometimes built into a pocket in the wall.

As the tees are more than 40 ft. long, there must be a joint in each. These can be in bay 21 for top flange and 31 for bottom flange. The sectional area cut through by the joint $=$ $4\frac{1}{4}$ sq. ins. A 6-in. by $\frac{3}{8}$-in. plate on top, and two 3-in. by $\frac{3}{8}$-in. plates on the web will give $4\frac{1}{2}$ sq. ins., but the tee will be reduced in strength in the tension flange by the end rivet holes through web. It will be seen from Fig. 167 that the stress in this bay is 30 tons, while the flange is designed for the stress of 35 tons which occurs in the adjoining bay so that it will bear the loss of the rivet hole.

CHAPTER XV

Travelling Cranes and Rolling Loads—Stress Diagrams for Rolling Loads—Lattice Stanchions

A simple case of a rolling load occurs when the girders for an overhead travelling crane at a power station are carried on piers against the side walls, as shown on the right-hand side of Fig. 174 a. Assume the load to be lifted to be 5 tons, the width of building between the centres of girders 35 ft., and the span from

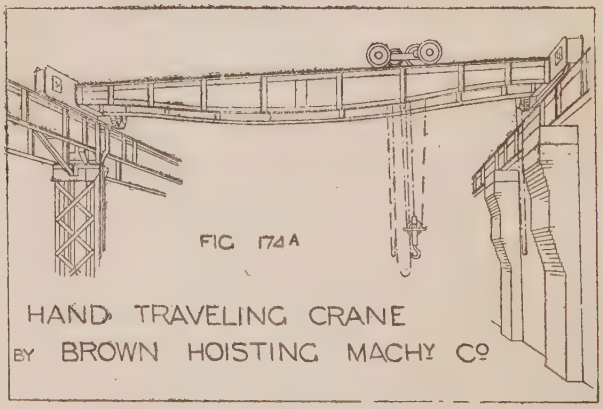


Fig. 174 a.—Sketch of a Hand Travelling Crane by the Brown Hoisting Machinery Co.

centre to centre of piers 16 ft. The lifting crab will weigh about ·75 ton, and the travelling girders carrying it about 3 tons, running on two wheels at each end about $\frac{1}{6}$ span, say 6 ft. centres. The surging of the load due to the slipping of the sling chain, or to the sudden starting of the crane in lifting, or stopping in lowering, will have a varying effect up to a maximum of double the load, but generally $1\frac{1}{2}$ times is sufficient to allow, and the 5-ton load will, therefore, be considered $7\frac{1}{2}$ tons. The crab

may be at any point of the span of the travelling girders, so that the whole load with the weight of crab and half that of the travelling girders will have to be taken as coming upon the wall

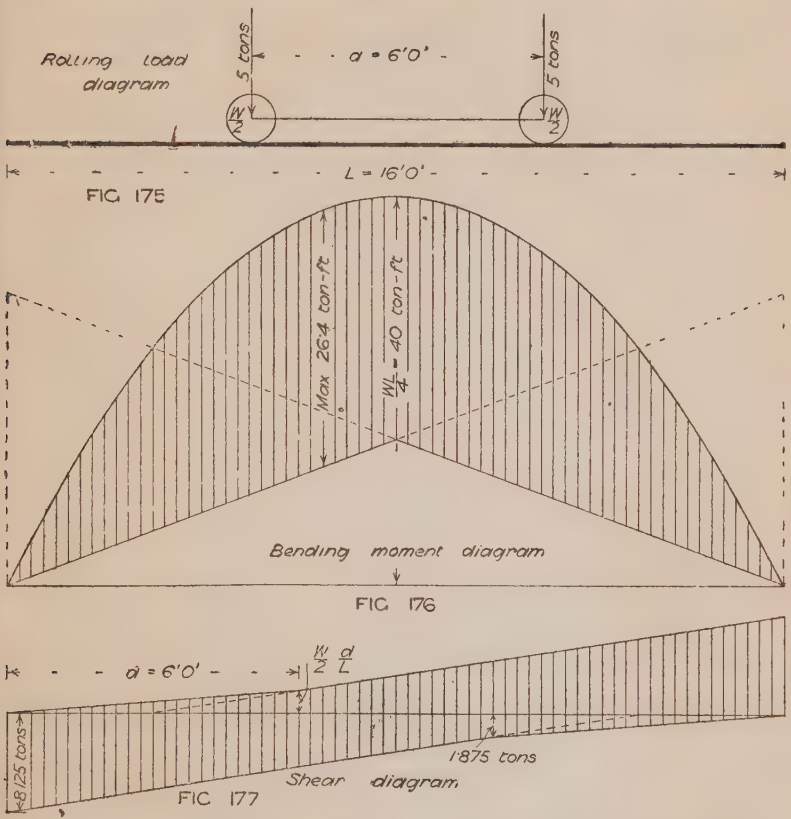


FIG. 175.—Rolling Load diagram for Girder carrying Travelling Crane.
FIG. 176.—Bending Moment diagram for ditto. FIG. 177.—Shear diagram for ditto.

girders. This will give, say, $7{\cdot}5 + {\cdot}75 + \frac{3}{2} = 9{\cdot}75$, say 10 tons at each end, or $\frac{10}{2} = 5$ tons on each wheel, and the resulting load diagram will be as Fig. 175. The bending moment diagram, to show the effect of the loads crossing the entire span, will then be constructed as shown in Fig. 176, and the shear diagram for the same complete passage of the loads will be as Fig. 177.

The maximum bending moment occurs at $\frac{l}{2} - \frac{d}{4} = \frac{16}{2} - \frac{6}{4} =$ 6·5 ft. from either support, and $= \frac{x}{l}\left\{W(l-x) - \frac{W}{2}d\right\} = \frac{6{\cdot}5}{16}$ $\left\{10(16 - 6{\cdot}5) - \frac{10}{2} \times 6\right\} = 26{\cdot}4$ ton-ft. The uniformly distributed load to produce this $= \frac{8M}{L} = \frac{8 \times 26{\cdot}4}{16} = 13{\cdot}2$ tons on a span of 16 ft. requiring a (B.S.B.21) 12-in. by 6-in. by 44-lb. rolled steel joist, which has a carrying capacity of 16 tons on this span. The extra bending moment due to the weight of girder may now be added to the bending moment diagram, the maximum ordinate in the centre being $\frac{wL^2}{8} = \frac{\frac{44}{2240} \times 16^2}{8} = 0{\cdot}63$ ton-ft., or 0·6 ton-ft. at the point of the span vertically under the previous maximum bending moment. The maximum bending moment would then be increased to 27 ton-ft., and the equivalent uniformly distributed load $= \frac{8M}{L} = \frac{8 \times 27}{16}$ $= 13{\cdot}5$ tons, so that the 12-in. by 6-in. by 44-lb. joist still has a good margin. The standard joint for this size joist is shown in Fig. 178, and this should always be arranged to come over a support, otherwise the lower flange would require to have a cover plate also.

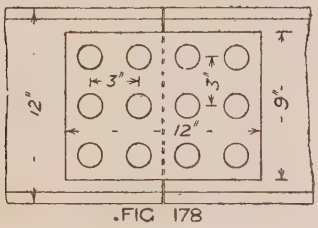


FIG. 178.—Standard Joint for Girder.

Lattice stanchions are sometimes adopted when not required to be covered in, the simplest being formed of two channel bars connected by plain tie plates, as shown in Fig. 179. When greater stiffness is required, diagonal lattice bars are used at 30 degs. or 45 degs. from the horizontal, as shown in Fig. 180. The lattice bars are usually flat, but when extra stiffness is required they may be of angle section. Double lattice bars as Fig. 181 are less often seen, and sometimes they are used with horizontal bars between, as Fig. 182. The plan Fig. 183 will be common to all these elevations. Lattice stanchions may consist

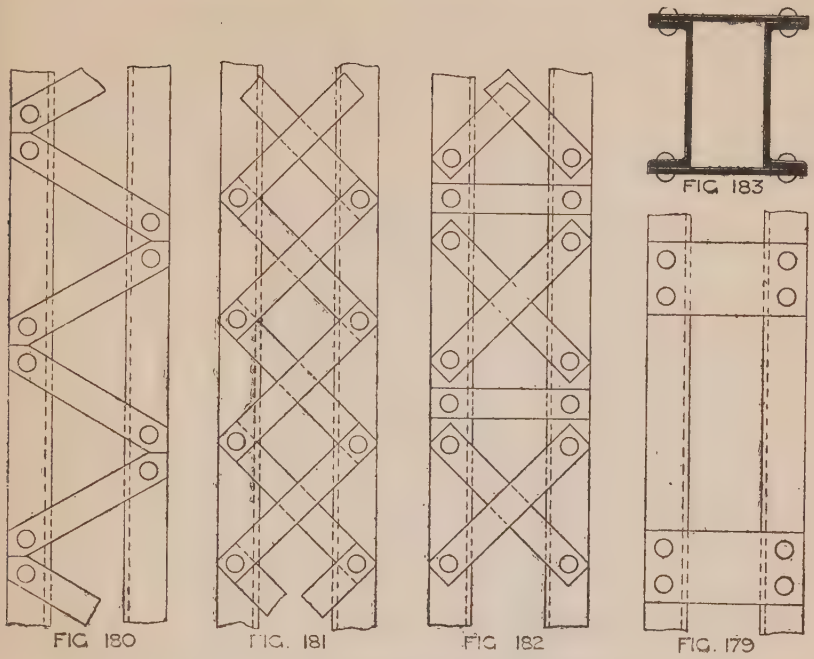


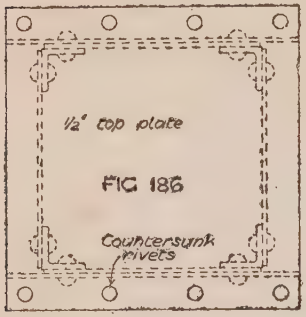
FIG. 179.—Lattice Stanchion of Channel Sections and Horizontal Strap Bars.
FIG. 180.—Lattice Stanchion with Lattice Bars at 30 degs. from horizontal.
FIG. 181.—Lattice Stanchion with Double Lattice Bars.
FIG. 182.—Lattice Stanchion with Double Lattice Bars and Horizontal Ties.
FIG. 183.—Section applying to Figs. 179 to 182.

of four angle bars, with lattices on all four sides, as in Figs. 184 and 185. The ends of lattice stanchions are generally finished off with a solid plate on each side, equal in depth to the width of stanchion, with an angle bar on each to make a flange to receive the top plate, as shown in Figs. 185 and 186, but they are not generally used for more than one storey, and the top should be kept small when a girder rests upon it to avoid the danger of eccentric loading. The bottom would be constructed in a similar manner with the

FIG. 184.—Section of Angle and Lattice-bar Stanchion.

detail according to the foundation it rests upon, as shown in Figs. 187 and 188.

Assume a lattice stanchion to be required, open all round, 18 ins. square, to carry 70 tons and to be 20 ft. high. For a trial section allow $5\frac{1}{2}$ tons per sq. in. compression on the angle bars, as the great width of stanchion will render it very stiff, then $\frac{70}{4 \times 5{\cdot}5} = 3{\cdot}18$ sq. in. sectional area of each. This is given by



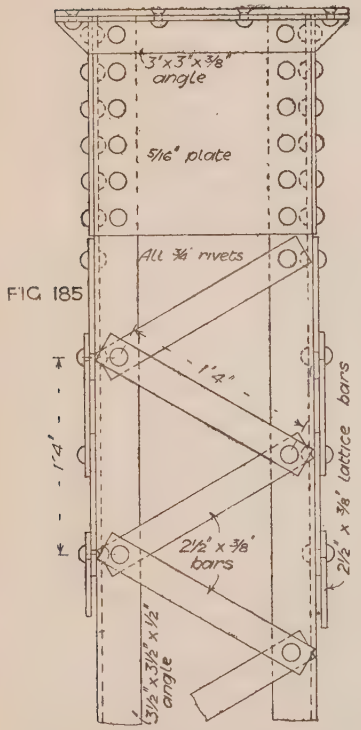


FIG. 185.—Elevation of Angle and Lattice-bar Stanchion.

FIG. 186.—Plan on top of ditto.

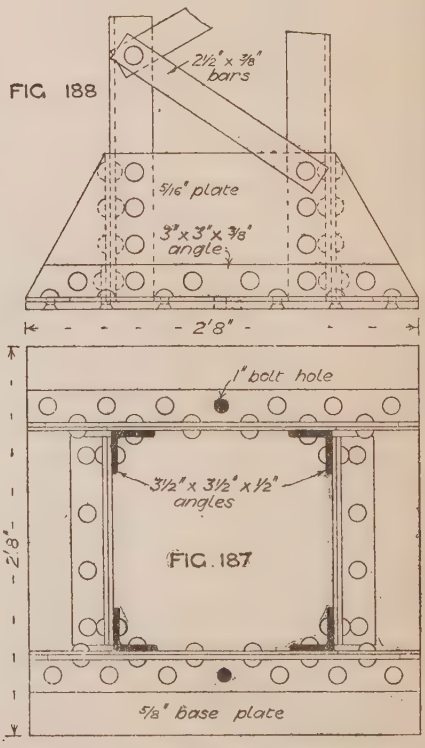


FIG. 187.—Plan of base of Angle and Lattice-bar Stanchion.

FIG. 188.—Elevation of base of ditto.

$3\frac{1}{2}$ in. $\times$ $3\frac{1}{2}$ in. $\times$ $\frac{1}{2}$ in. angle, which equals 3·25 sq. in. Taking the stanchion as a whole, and assuming both ends fixed, the safe stress by Gordon's formula will be

$$W = \frac{f}{1 + \frac{m}{nq}\left(\frac{l}{d}\right)^2} = \frac{6\frac{1}{2}}{1 + \frac{1}{\frac{1}{2} \times 3{,}750}\left(\frac{20 \times 12}{18}\right)^2} = 5{\cdot}91 \text{ tons}$$

per sq. in., and the total safe load will be 5·91 $\times$ 3·25 $\times$ 4 = 76·8 tons, which leaves a small margin above the load to be carried. The strength of the unsupported length of one angle, assuming both ends fixed, and the lattice bars 16 ins. apart, must also be taken by Gordon's formula, then

$$W = \frac{f}{1 + \frac{m}{nq}\left(\frac{l}{d}\right)^2} = \frac{6\frac{1}{2}}{1 + \frac{1}{\frac{1}{2} \times 3{,}750}\left(\frac{20}{2{\cdot}8}\right)^2} = 6{\cdot}31 \text{ tons}$$

per sq. in. safe stress, and the total safe load on each angle 6·31 $\times$ 3·25 = 20·50 tons, whereas the load to be carried by each angle $= \frac{70}{4} = 17{\cdot}5$ tons, so that there is a good margin, and the lattice bars might have been placed at a steeper angle to leave a longer portion of angle bar unsupported. Redpath, Brown & Co. give a formula for calculating the spacing of the diagonal bars in lattice stanchions, as follows :—

L = height of stanchion in inches.
R = radius of gyration for one channel.
S = maximum spacing of lattice bars in inches.

$$S = \frac{R^2 \, min. \times L^2}{2 \, R^2 \, max.}$$

and state that the spacing given by this formula is much in excess of that adopted in practice. It is, in fact, so much in excess applied to ordinary cases that it is difficult to see where its utility comes in, and we cannot use it here. G. L. Bilderbeck gives a formula for stresses in lacing bars in "Engineering Record," but the information accompanying it is insufficient to enable it to be used, so that until some reasonable formula is forthcoming the lattice bars must be designed by personal judgment. As a maximum they would be made equal in section to one side of the angle bars, or to one flange of a channel bar,

but probably half of this sectional area would be sufficient in ordinary circumstances. In the present case $3\frac{1}{2}$ ins. by $\frac{1}{2}$ in. would be the maximum, and, say, $2\frac{1}{2}$ ins. by $\frac{3}{8}$ in. the more suitable size. A $3\frac{1}{2}$-in. by $\frac{1}{2}$-in. bar minus one $\frac{3}{4}$-in. rivet would have a sectional area of 1·375 sq. in., which at $6\frac{1}{2}$ tons per sq. in. would allow for a tension of $1{\cdot}375 \times 6{\cdot}5 = 8{\cdot}94$ tons. A $\frac{3}{4}$-in. rivet in single shear is worth 2·2 tons, so that, to give equal strength in all parts, with $3\frac{1}{2}$-in. by $\frac{1}{2}$-in. bars, $\frac{8{\cdot}94}{2{\cdot}2} = 4$ rivets, would be required in each end, necessitating gusset or joint plates. A $2\frac{1}{2}$-in. by $\frac{3}{8}$-in. bar, less one $\frac{3}{4}$-in. rivet, would have a sectional area of ·66 sq. in., which at $6\frac{1}{2}$ tons per sq. in. would allow for a tension $·66 \times 6{\cdot}5 = 4{\cdot}29$ tons, then $\frac{4{\cdot}29}{2{\cdot}2} =$ say 2 rivets in each end. This could not be done without small gusset plates being added which it is desirable to avoid, and it is useless to make the bars themselves stronger than their connections. These bars might, however, be in compression, and a $2\frac{1}{2}$-in. by $\frac{3}{8}$-in. bar, 1 ft. 4 ins. long, would only resist

$$\frac{2\frac{1}{2} \times \frac{3}{8} \times 6{\cdot}5}{1 + \frac{2{\cdot}5}{\frac{1}{2} \times 3{,}750}\left(\frac{16}{\frac{3}{8}}\right)^2} = 1{\cdot}78 \text{ tons},$$

if taken as equivalent to one end fixed and one rounded, and one $\frac{3}{4}$ in. rivet will be sufficient.

CHAPTER XVI

Cast-iron Stanchions and their Safe Loads—Cast-iron Hollow Columns and their Safe Loads—London Building Act Amendments, 1909

Cast-iron stanchions or story posts are still used where foundry work is cheap, especially in small warehouses, and mills where attachments are required for countershafts. For large warehouses and buildings of several floors they are, however, not so good as mild steel stanchions, particularly where steel girders are used in the floors, as they are not so readily connected to secure the stability of the building. The behaviour of the two materials in a fire is instructive. Cast-iron remains rigid, and resists the effect of fire until it reaches practically a melting temperature, but if water is played on it while red hot it is very apt to crack and fly. Mild steel, on the other hand, loses its strength and bends as soon as it gets red hot, dragging down the floors and wrecking the whole building. It is very desirable that all structural ironwork should be protected from fire, especially where the stability of the building depends upon it.

The calculation of cast-iron stanchions is very simple, and a small example may be taken to illustrate the method. A + or H section is used when the stanchion is detached, and a channel or ⊔ section when against a wall. Assume a detached stanchion of plain + section, 10 ft. high, to carry a load of 25 tons. The width across the arms should be between $\frac{1}{12}$th and $\frac{1}{20}$th of the length, and the thickness of metal from $\frac{1}{6}$th to $\frac{1}{10}$th of the width. As the load is rather small let the width be $\frac{1}{16}$th length $= 7\frac{1}{2}$ ins., and the thickness, say $\frac{3}{4}$ in.; the section will then be as Fig. 189, which shows the engineer's section, or Fig. 190, which shows the architect's section; but the latter has many objectionable features.

It is most desirable in cast-iron work to have all angles rounded, especially internal angles, as the crystals in cooling arrange themselves perpendicular to the surface at which the heat escapes, and any square angle gives a bad mitre of the crystals, and encourages fracture at their junction.

The width over the arms will not be the "least width," as, with $7\frac{1}{2}$ ins. over the arms, the least width will be $\sqrt{\frac{7{\cdot}5^2}{2}} = \frac{7{\cdot}5}{\sqrt{2}} = \frac{7{\cdot}5}{1{\cdot}414} = 5{\cdot}3$ ins., and the sectional area, allowing the material added by the internal curves to equal the material

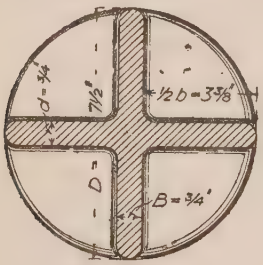


FIG. 189

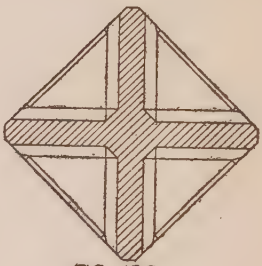


FIG. 190

FIG. 189.—Engineer's Section of Cast-iron + Stanchion.
FIG. 190.—Architect's Section of + Stanchion.

lost by the external curves, equals $2\ (7{\cdot}5 \times {\cdot}75) - {\cdot}75^2 = 10{\cdot}69$ sq. ins. The following table gives the approximate safe load in tons per sq. in. upon cast-iron stanchions.

SAFE LOAD ON CAST-IRON STANCHIONS.

Up to	8	diameters long	= 5	tons	per sq. in.
,,	10	,,	= 4	,,	,,
,,	13	,,	= 3	,,	,,
,,	15	,,	= $2\frac{1}{2}$	,,	,,
,,	18	,,	= 2	,,	,,
,,	24	,,	= $1\frac{1}{2}$	,,	,,
,,	32	,,	= 1	,,	,,
,,	40	,,	= $\frac{3}{4}$	,,	,,
,,	50	,,	= $\frac{1}{2}$	,,	,,

The ratio of length to diameter over arms being 16, the safe load will be, say, 2·3 tons per sq. in.; then the sectional area being 10·69 sq. ins., the safe load will be $10{\cdot}69 \times 2{\cdot}3 = 24{\cdot}94$, say 25 tons. If the safe load had not come out at the desired amount the thickness of arms would be altered to adjust the sectional area to give the safe load required. The actual strength of the stanchion of these dimensions must now be checked by Gordon's formula, allowing a factor of safety of 6. Had the stanchion been over 25 diameters long a factor of safety of 10 would have been desirable.

The total breaking weight by Gordon's formula =

$$p = \frac{fA}{1 + \frac{m}{nq}\left(\frac{l}{d}\right)^2} = \frac{36 \times 10{\cdot}69}{1 + \frac{1}{\frac{1}{2} \times 750}\left(\frac{10 \times 12}{5{\cdot}3}\right)^2} = 162{\cdot}4 \text{ tons},$$

and, allowing a factor of safety of 6, the safe load will be $\frac{162{\cdot}4}{6}$ = say, 27 tons, so that the section adopted will be just sufficient.

When a stanchion is designed for the full load applied axially, and only sufficient metal provided for that purpose, there will be considerable risk of failure from unsymmetrical loading, even by a comparatively small portion of the load, when it is superimposed on one side of the stanchion. In the present case the stanchion will be stressed to its full working capacity by a load on one side of only 1·76 tons. This is found as follows.

The working limit is 2·3̇ tons per sq. in., therefore $\frac{W}{A} + \frac{M}{Z}$ = 2·3̇. The area = 10·69 sq. ins., the bending moment = $W \times \frac{10 + 7\frac{1}{2}}{2}$ (See Fig. 191) = $8{\cdot}75W$, and the section modulus = $\frac{BD^3 + bd^3}{6\,D}$ (See Fig. 189) = $\frac{\frac{3}{4} \times (7\frac{1}{2})^3 + 6\frac{3}{4} \times (\frac{3}{4})^3}{6 \times 7\frac{1}{2}} = 7{\cdot}1$ ins. units. Then $\frac{W}{10{\cdot}69} + \frac{8{\cdot}75W}{7{\cdot}1} = 2{\cdot}\dot{3}$, or $\cdot 093W + 1{\cdot}23W = 2{\cdot}\dot{3}$, whence $1{\cdot}323\ W = 2{\cdot}\dot{3}$, or $W = 1{\cdot}76$ tons. The half load that might be expected on one side of the stanchion if the floor on that side were fully loaded would be $\frac{25}{2} = 12\frac{1}{2}$ tons, whereas, from the above calculation, it will be seen that only 1·76 tons

could be safely so carried. This is a very serious matter, and indicates a source of danger that few people are aware of.

The weight of stanchion being, say, $\frac{1}{4}$ ton, the total load to be carried by the base will be $25 + \frac{1}{4} = 25{\cdot}25$ tons. Assuming the cast iron base plate to rest on 1 : 4 : 8 concrete capable of bearing 9 tons per square ft., the area of base plate required will be $\frac{25{\cdot}25}{9} = 2{\cdot}8$ sq. ft., or, say, 1 ft. 8 ins. square. It is generally advisable to make up for the section removed by the bolt holes in the base plate by adding a raised boss, as shown in Fig. 196, so that the section across the plate through the hole remains of the full original amount.

A distributed load is transmitted from the cap and base plate most directly by plain hollow curves, as shown in Fig. 191, but for architectural effect the outline is sometimes varied, as shown in Figs. 192 to 195. The elevation of the stanchion may now be drawn out as Fig. 191, plan of base as Fig. 196, and plan on top as Fig. 197, which shows the socket for stanchion on upper floor and two side supports for floor girders.

An example may now be taken of a hollow cylindrical cast-iron column 20 ft. high to carry 20 tons. The external diameter of the column will often not exceed $\frac{1}{10}$th of the length, and should not be less than $\frac{1}{35}$th in extreme cases. The thickness of metal may vary from $\frac{1}{8}$th to $\frac{1}{16}$th of the external diameter, but as a general rule columns of 6 ins. diameter and over should be not less than $\frac{3}{4}$ in. thick, and under 6 ins. diameter not less than $\frac{1}{2}$ in. thick, but in London the building regulations do not permit of any less thickness than $\frac{3}{4}$ in. for hollow cast iron columns, or $\frac{1}{12}$th diameter, whichever is the greater.

For architectural effect, with a modicum of scientific basis, circular columns are generally increased in diameter towards the bottom, but the least diameter is taken into account in the calculations of strength, although probably the diameter halfway down would give a result nearer to the actual strength. Assume the column to be $7\frac{1}{2}$ ins. diameter at the top, with $\frac{3}{4}$ in. thickness of metal, then the area $= \frac{\pi}{4}(D^2 - d^2) = \frac{3{\cdot}1416}{4}(7{\cdot}5^2 - 6^2) = 15{\cdot}9$ sq. ins. The following table gives

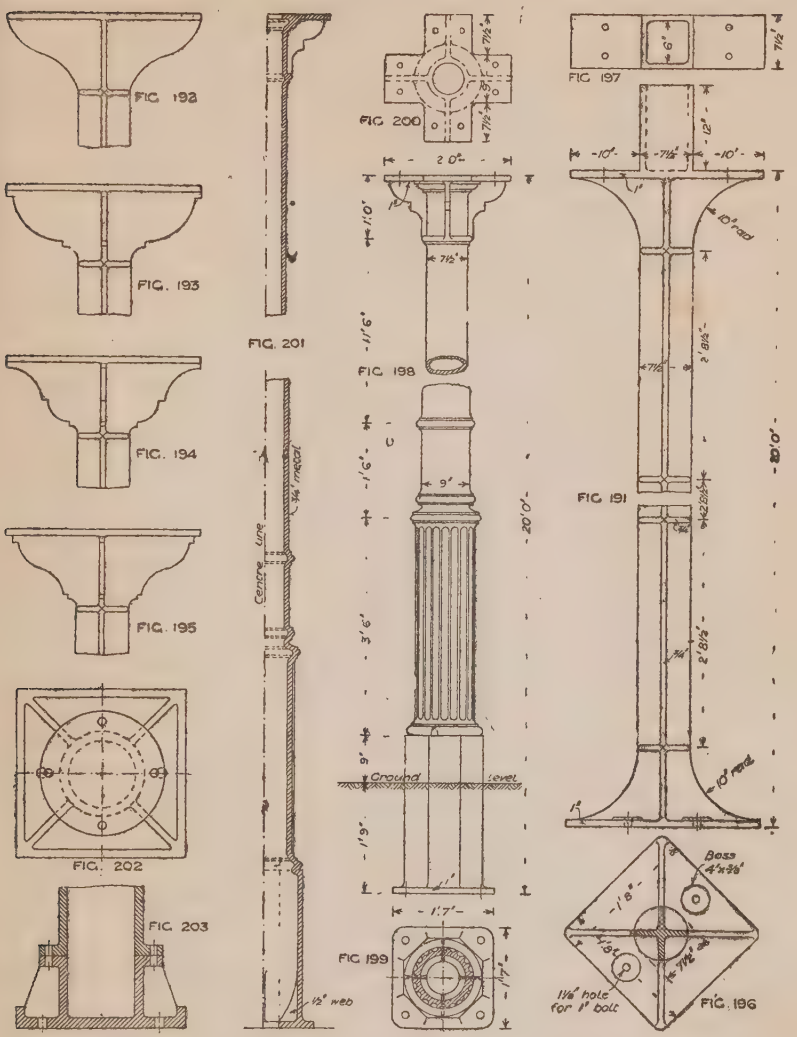


FIG. 191.—Elevation of Stanchion. FIGS. 192 to 195.—Methods of Finishing Heads of Stanchions. FIG. 196.—Plan of Base of Stanchion. FIG. 197.—Plan on Top of Stanchion. FIG. 198.—Elevation of Hollow Circular Cast-iron Column. FIG. 199.—Plan of Base of Column. FIG. 200.—Plan on Top of Column. FIG. 201.—Half-section through Column. FIG. 202.—Plan of Base of Column in two parts. FIG. 203.—Section of Base Column in two parts.

the approximate safe load in tons per sq. in. upon hollow cast-iron columns :—

SAFE LOAD ON HOLLOW CAST-IRON COLUMNS.

For a thickness of $\frac{1}{12}$th diameter :—

Up to 10 diameters long	=	5	tons	per	sq. in.
10 „ 15 „ „	=	4	„	„	
15 „ 20 „ „	=	3	„	„	
20 „ 25 „ „	=	2	„	„	
25 „ 30 „ „	=	$1\frac{1}{2}$	„	„	
30 „ 35 „ „	=	$\frac{3}{4}$	„	„	

In the present case the column will be $\frac{20 \times 12}{7{\cdot}5} = 32$ diameters long, and by interpolation from the above table the safe load will be, say, 1·25 tons per square inch, and the area being 15·9 sq. in., the total approximate safe load $= 15{\cdot}9 \times 1{\cdot}25 = 19{\cdot}88$, say 20 tons, but the actual safe load will probably be considerably higher, as approximate rules should always be well on the safe side.

The safe load for a column of these dimensions may now be worked out by Gordon's formula; then

$$p = \frac{fA}{1 + \frac{m}{nq}\left(\frac{l}{d}\right)^2} = \frac{36 \times 15{\cdot}9}{1 + \frac{1}{1\frac{1}{2} \times 750}\left(\frac{20 \times 12}{7{\cdot}5}\right)^2} = 300 \text{ tons};$$

the factor of safety in this case may be taken as 10, and the safe load will be $\frac{300}{10} = 30$ tons, which gives a good margin.

By the London Building Act Amendments of 1909 cast-iron pillars with both ends fixed may have a working stress not exceeding that given in the table on p. 135.

The radius of gyration $r = \sqrt{\frac{I}{A}}$ and for a hollow cylindrical section $I = \frac{\pi}{64}(D^4 - d^4)$, while $A = \frac{\pi}{4}(D^2 - d^2)$, therefore $\frac{I}{A}$ $= \frac{\frac{\pi}{64}(D^4 - d^4)}{\frac{\pi}{4}(D^2 - d^2)} = \frac{1}{16}(D^2 + d^2)$, which for this column $=$ $\frac{1}{16}(7{\cdot}5^2 + 6^2) = \frac{56{\cdot}25 + 36}{16} = 5{\cdot}76$, whence $r = \sqrt{5{\cdot}76} = 2{\cdot}4$,

CAST-IRON PILLARS

Ratio of Length to Least Radius of Gyration.	Working Stresses in Tons per sq. in.
20	4·5
30	4·0
40	3·5
50	3·0
60	2·5
70	2·0
80	1·5

and the ratio of length to least radius of gyration $= \frac{20 \times 12}{2{\cdot}4} = 100$, which is beyond the limits of the table; but, following in the same ratio, the allowable working stress will be, say, 0·5 ton per square inch. The sectional area of the column is 15·9 sq. in., then $15{\cdot}9 \times 0{\cdot}5 = 7{\cdot}95$, say 8 tons, safe working load. When these regulations were being drawn up the London County Council were charged with requiring an excessive amount of material in the various parts of a structure, and this illustration partly confirms the view stated, but when the risks of unequal loading are considered, perhaps the London County Council are not so far out. To conform to their regulations the thickness of metal would apparently have to be increased, but this cannot be determined directly, as the radius of gyration will alter with the thickness, and trial must be made. Try 1 in. thick, then

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{1}{16}(7{\cdot}5^2 + 5{\cdot}5^2)} = \sqrt{5{\cdot}4} = 2{\cdot}32, \frac{20 \times 12}{2{\cdot}32} = 103$$

for ratio of length to radius of gyration, making the safe load = say, 0·4 ton per square in. The area = ·7854 $(7{\cdot}5^2 - 5{\cdot}5^2) =$ 20·42 sq. ins., $20{\cdot}42 \times 0{\cdot}4 = 8{\cdot}17$ tons. This slight gain is due to the increased thickness reducing the radius of gyration and the extreme case would be to make the column solid, when the radius of gyration would become $\sqrt{\frac{1}{16}D^2} = \frac{1}{4}D = \frac{1}{4} \times 7{\cdot}5 =$ 1·875. Then the ratio of length to radius of gyration will be $\frac{20 \times 12}{1{\cdot}875} = 128$, which runs beyond the range at which the

table can be made to apply, making it appear that a solid $7\frac{1}{2}$ in. column is weaker than one of the same external diameter and $\frac{3}{4}$ in. thickness. In other words, the thinner the column the greater the stress per sq. in. which may be put upon it. It will thus be seen that to conform to the London County Council regulations there is no alternative but to increase the diameter. Try $8\frac{1}{2}$ ins. diameter and $\frac{3}{4}$ in. thickness, then

$r = \sqrt{\frac{1}{16}(8{\cdot}5^2 + 7^2)} = 2{\cdot}75$, and the ratio $\frac{l}{r} = \frac{20 \times 12}{2{\cdot}75} = 87{\cdot}2$, which gives an allowable stress of, say, 1·14 tons per sq. in. The area $= {\cdot}7854\ (8{\cdot}5^2 - 7^2) = 18{\cdot}26$ sq. in.; therefore the total safe load $= 18{\cdot}26 \times 1{\cdot}14 = 20{\cdot}82$ tons; so that this section would be just sufficient.

The weight of the column will be approximately $\frac{3}{4}$ ton; so that the total load will be 20·75 tons, and, assuming the base to rest on 1 : 4 : 8 concrete, as before, the area of base plate required will be $\frac{20{\cdot}75}{9} = 2{\cdot}3$ sq. ft., and, allowing for loss of area from a 6 in. diameter hole in the centre, will require to be, say, 1 ft. 7 ins. square. The elevation of the column will be as in Fig. 198, plan of base of column as Fig. 199, and plan on top of column as Fig. 200. It should be noted that when flutes or mouldings are used on a column the decorations must be extra over the thickness of metal required to take the load, and the metal must as far as possible be kept of uniform thickness, as shown in the section Fig. 201.

The base plate may be detached from the column and bolted on, with machined faces as shown in Figs. 202 and 203, particularly in the case of large columns. The outer edge of base plate may be stiffened by a raised rib, as shown in these figures.

Stanchions and columns should be cast vertically with a "sullage head," that may be cut off after the metal is cold. This is to ensure the absence of honey-combing in the casting, from bubbles of gas rising to the higher side when cast horizontal. Ironfounders generally claim that an angle of 30 degs. complies with this specification, on the plea that if it is not horizontal it is *more or less* vertical.

CHAPTER XVII

Designing Cast-iron Cantilevers and Girders under Various Loading—Girders for Carrying Machinery

Cantilevers of wrought or cast-iron, or pieces of rolled joist used as cantilevers, are commonly employed to support galleries and balconies. There is no difficulty in designing them, but the fixing involves some interesting points. Assume that cast-iron cantilevers, projecting 6 ft. from an 18-in. brick wall are required at intervals of 10 ft. to carry a distributed load of $1\frac{1}{2}$ cwts. per ft. super., which will be sufficient to include the structural load. The load on each will be $6 \times 10 \times 1\frac{1}{2} = 90$ cwts. $= 4{\cdot}5$ tons. The load diagram will be as Fig. 204, and the bending moment diagram as on the right of Fig. 205. The bending moment diagram follows the construction for a cantilever with a distributed load over part of its length, the bending moment at *A* (Fig. 204) will be $4{\cdot}5 \times (\frac{6}{2} + {\cdot}25) = 14{\cdot}625$ ton-ft., set up at *a b* Fig. 205. Join *b* to *c*, which is the centre of the length occupied by the distributed load, and from the inner face of wall *d* set up a perpendicular *d e* to meet *b c* in *e*. Then construct a semi-parabola with a length *d f* and depth *d e*, giving the complete bending moment diagram as *a b e f*. Cast-iron is six times as strong in compression as in tension, and theoretically the flanges should have a sectional area inversely proportioned to the strength of the material; but to allow for practical contingencies such as honeycombing in the small flange, it is usual to make the sectional areas in the ratio of 1 to 4. The tension flange alone is subject to calculation, the working stress allowed for dead loads being $1\frac{1}{2}$ tons per sq. in. In this case the maximum bending moment is 14·625 ton-ft., and if the total depth be made 12 ins. the mean depth will be, say 11 ins. and the sectional area required for the bottom flange will be $\frac{14{\cdot}625}{\frac{11}{12} \times 1\frac{1}{2}}$ $= 10{\cdot}6$ sq. ins., and, as the thickness may be from $\frac{1}{8}$th to $\frac{1}{12}$th

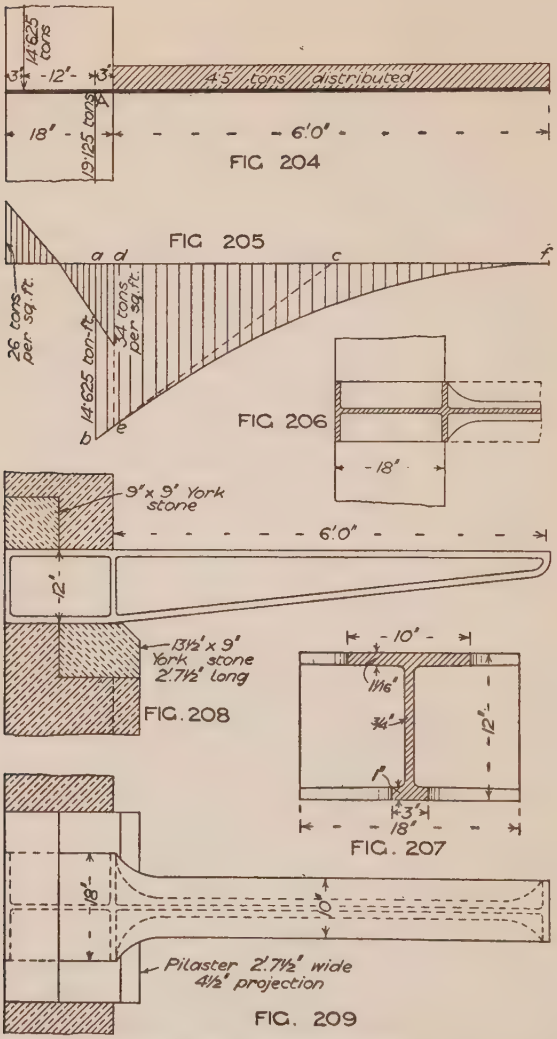


FIG. 204.—Load Diagram for Cantilever, showing the reactions on the Wall. FIG. 205.—Bending Moment Diagram for Fig. 204, showing the Distribution of Stress over the Wall. FIG. 206.—Sectional Plan of Cast-iron Cantilever. FIG. 207.—Cross Section of Cantilever with Wide Flanges. FIG. 208.—Elevation of Cantilever. FIG. 209.—Plan of Cantilever.

of the width, it will be, say, 10 ins. by $1\frac{1}{16}$ ins. To enable the pattern to be drawn out of the sand in moulding, the flange may

be tapered $\frac{1}{8}$ in. on the inside, making the thickness at the edges 1 in. and in the centre $1\frac{1}{8}$ ins. The compression flange, with a sectional area of one-fourth the tension flange, will contain 2·65 sq. ins., and a suitable size would be, say, 3 ins. by 1 in. The web, which takes all the shear, is not proportioned to the shearing stress, as there will always be ample material. The consideration which determines the thickness of web is the thickness of flanges, as it is essential in a casting, while keeping every part as light as possible, to have the thickness fairly uniform throughout, to obtain an equal rate of cooling and avoid initial stresses. Generally the web is made $\frac{2}{3}$ to $\frac{3}{4}$ the thickness of tension flange, say $\frac{3}{4}$ in. The internal angles of a casting should have curved fillets of a radius equal to $\frac{1}{2}$ or $\frac{2}{3}$ of the parts joined, say, in this case, all $\frac{1}{2}$-in. radius.

The end built in the wall should be formed like a box, to give stiffness to the cantilever at the part where the greatest stress comes, and to give a wide bearing surface. A sectional plan on bottom is shown in Fig. 206, the dotted lines indicating the tension flange. Cantilevers are very often improperly fixed, because the principles are not understood, and it is more by good luck than good management that failures do not more often occur. It will be seen, on looking at the bending moment diagram, where the pressures on the wall are shown to the left of the bending moments, that the cantilever tends to rotate upon the centre of the wall, the pressures varying from zero at the centre to a maximum in opposite directions at each outer face. The pressures will vary as ordinates to triangles, and the centre of pressure will therefore be at one-sixth of the thickness from each face. The pressures will be calculated by leverage as follows : 4·5 tons load × 3 ft. 3 ins. leverage to centre of pressure at front, divided by 1 ft. leverage to centre of pressure at back, equals 14·625 tons upward pressure at back, then 4·5 tons on one side and 14·625 tons on the other give a total reaction or downward pressure near inner face of wall of 19·125 tons. It is possible that this is not the exact distribution of stress, but it will be approximately correct. When the pressures vary as ordinates to a triangle the maximum pressure will be double the mean, and therefore, with a total of 19·125 tons spread over ·75 ft. × $\frac{10}{12}$ ft.,

the maximum pressure will be $\frac{2 \times 19{\cdot}125}{{\cdot}75 \times \frac{10}{12}} = 61{\cdot}2$ tons per sq. ft. This shows that it will be desirable to considerably widen the wall-hold of the cantilever, and it may be increased to 18 ins., as shown in the section Fig. 207. The maximum pressure will then be $\frac{2 \times 19{\cdot}125}{1{\cdot}5 \times {\cdot}75} = 34$ tons per sq. ft., which is still a very high pressure for York stone, but may be permissible with selected material of good thickness. The front template should, therefore, not be less than 2 ft. 6 ins. long by 12 ins. wide by 9 ins. thick, the extra 3 ins. width being allowed to project, with chamfers on the top and bottom edges, or, better still, made 2 ft. $7\frac{1}{2}$ ins. long and $13\frac{1}{2}$ ins. wide, chamfered 3 ins. on top and supported by a 2 ft. $7\frac{1}{2}$ ins. by $4\frac{1}{2}$ ins. pilaster below, as shown in elevation Fig. 208 and plan Fig. 209. The top-bearing stone, having less pressure to resist, may be made 2 ft. 6 ins. long by 9 ins. wide and 9 ins. thick. The theoretical outline for the elevation of a cantilever for a distributed load would follow the bending moment diagram, but practical considerations call for a modification as shown by Fig. 208. When a piece of rolled steel joist is used as a cantilever, other pieces of rolled joist may be used instead of stone templates to distribute the pressure along the wall, and where there is an insufficient weight of brickwork above the built-in end, holding-down bolts and plates must be attached and built in the brickwork sufficiently low down to obtain a secure hold.

A cast-iron parallel flange girder to carry a distributed load is a very simple matter. The sectional area of the bottom flange in sq. ins. may be made equal to the distributed load in tons, the mean depth one-twelfth of the effective span, or the total depth one-tenth of the clear span, the top flange one-fourth the area of bottom flange, and feathers or stiffeners about every 4 ft. Suppose a girder of this kind to be required for a clear span of 12 ft. 6 ins. to carry a load of ten tons, the girder may then be drawn out from these particulars, as shown in Fig. 210 elevation, Fig. 211 plan, and Fig. 212 section. The calculations will be total depth $= \frac{1}{10}$ clear span $= \frac{12{\cdot}5}{10} = 1{\cdot}25$ ft. $= 15$ ins. The

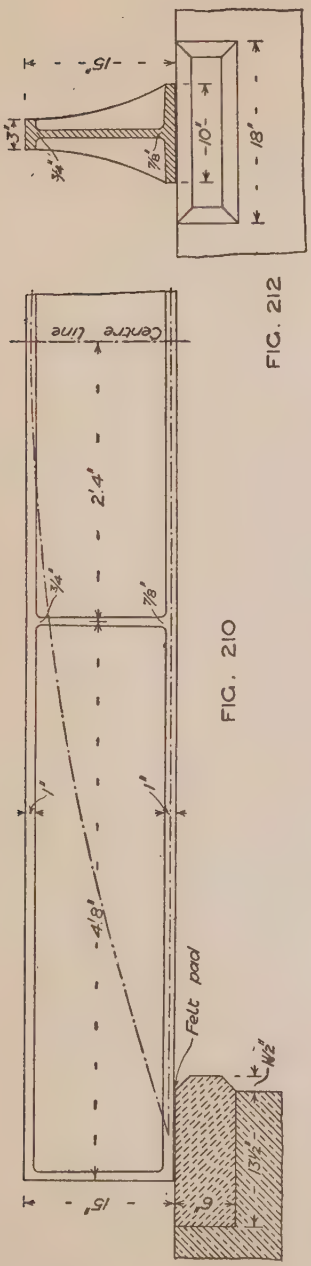


FIG. 210.—Part Elevation of Cast-iron Parallel Flange Girder supported at ends. FIG. 211.—Part plan of ditto.
FIG. 212.—Cross Section of ditto.

bottom flange will have a sectional area equal to the distributed load of 10 tons = 10 sq. ins., say 10 ins. × 1 in. average thickness, area of top flange = $\frac{10}{4}$ = 2·5 sq. ins., or, say 3 ins. × 1 in., and the web may be $\frac{7}{8}$ in. thick at bottom, tapering to $\frac{3}{4}$ in. thick at top. Allowing 12 tons per sq. ft. on the York stone, the girder will require a bearing area of $\frac{\frac{1}{2} \times 10}{12} = 0{\cdot}416$ sq. ft., or a length of $\frac{0{\cdot}416}{0{\cdot}83} = 0{\cdot}5$ft. = 6 ins., but should be made at least 9 ins. in order to throw the weight well within the face of the wall. Taking the safe load on the brickwork at 3 tons per sq. ft., the area of stone required = $\frac{\frac{1}{2} \times 10}{3} = 1{\cdot}6$ sq. ft., or, say, 18 ins. by 13½ ins. by 6 ins. Generally the stone is made to project 6 ins. beyond the girder on the sides and back = 22 ins. by 15 ins., and thickness one-third of the longest dimension = 7½ ins., instead of calculating it for safe load. The parabola shown by stroke and dot line on Fig. 210 indicates the variation of mean depth that might have been made to maintain uniform strength, but it is often more convenient to make the girder parallel throughout and waste the metal of the web that occurs above the outer ends of the parabola.

Girders of cast iron are much used in the framing to support sewage pumps and other machinery, but where machinery is concerned the actual load must be doubled to allow for the effect of vibration, and where shocks occur the depth must be increased to provide extra stiffness. Assume that a pair of cast iron girders of uniform strength and breadth are required to carry the bearings of a crank shaft, giving a central load of 2½ tons, which × 2 for vibration = 5 tons on each, over a 10-ft. span. The bending moment diagram for a central load is a triangle of base equal to the distance from centre to centre of bearing surfaces, and of a depth in centre equal to $\frac{Wl}{4} = \frac{2\frac{1}{2} \times 2 \times 10}{4} = 12{\cdot}5$ ton-ft., but the actual mean depth of girder may be $\frac{1}{12}$ span = $\frac{1}{12} \times 10$ ft. = 10 ins.; then the bending moment diagram should be drawn to such a scale that it represents the required depth of girder, in this case 1·25 ton-ft. to each one inch of depth of girder,

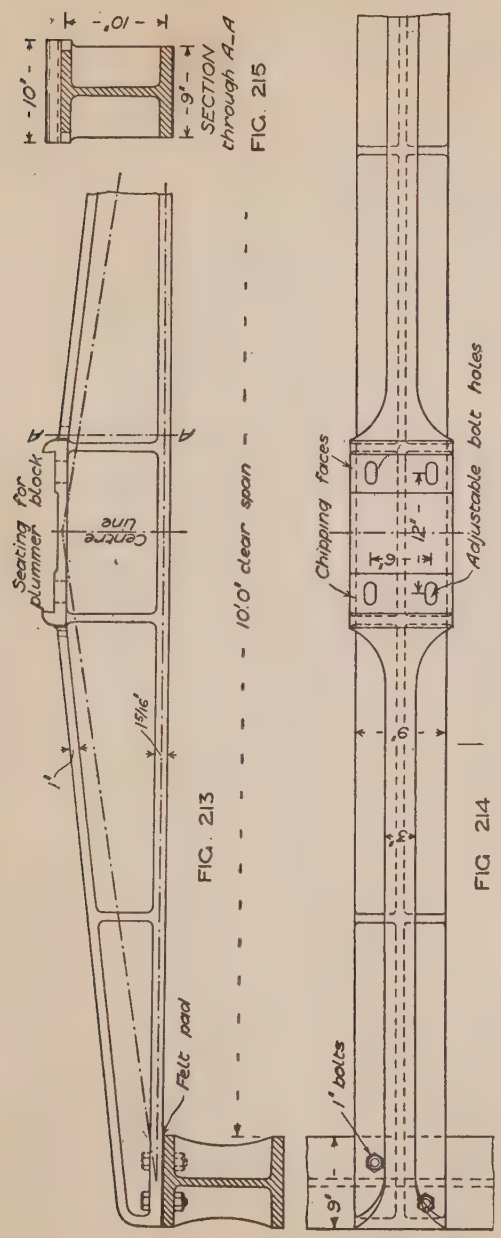


Fig. 213.—Part Elevation of Cast-iron Girder supported at ends to carry Concentrated Load. Fig. 214.—Part plan of ditto.
Fig. 215.—Cross Section of ditto.

according to the scale to which it is drawn. Approximate weight of girder $\frac{Wl}{125} = \frac{5 \times 10}{125} = \cdot 4$ ton. Width of bearing surface, say, 9 ins., upon a cross girder; effective span, 10 ft. 9 ins.; stress in centre $= \frac{Wl}{4\,d} = \frac{5{\cdot}4 \times 10{\cdot}75}{4 \times {\cdot}83} = 17{\cdot}41$ tons; working stress in tension, $1\frac{1}{2}$ tons per sq. in.; sectional area of bottom flange $\frac{17{\cdot}41}{1{\cdot}5} = 11{\cdot}6$ sq. ins.; width of flange, say, 9 ins.; thickness of flange $\frac{11{\cdot}6}{9} = 1{\cdot}29$ say $1\frac{5}{16}$ ins. Size of top flange $\frac{1}{4}$ area of bottom $= \frac{11{\cdot}6}{4} = 2{\cdot}9$ sq. ins., say, 3 ins. by 1 in.; thickness of web $\frac{3}{4}$ in. to $\frac{7}{8}$ in., and curves in internal angles, $\frac{3}{4}$ in. radius. The girder may now be drawn out as shown in elevation, Fig. 213, plan Fig. 214, and section through A—A, Fig. 215. The part where the load is applied must be designed to suit the particular circumstances of the case. Generally there will be a rectangular widening of the top flange large enough to take the base of plummer block, with raised "chipping faces," to allow of a "metal to metal" bearing with the minimum of machined surfaces; also with slot bolt holes to allow for adjusting the plummer block, and raised undercut ends to permit of wedges being inserted to keep the whole firm when correctly adjusted.

CHAPTER XVIII

Compound Girders—Box Girders—Shear Stress and Pitch of Rivets

For use in building construction, large girders should, as far as possible, be made up of rolled joists with flange plates at top and bottom, but there are many cases where it is necessary to build up a girder entirely of plates and angles.

An example of a rolled joist compound girder may be taken first, but instead of designing one to meet given conditions we

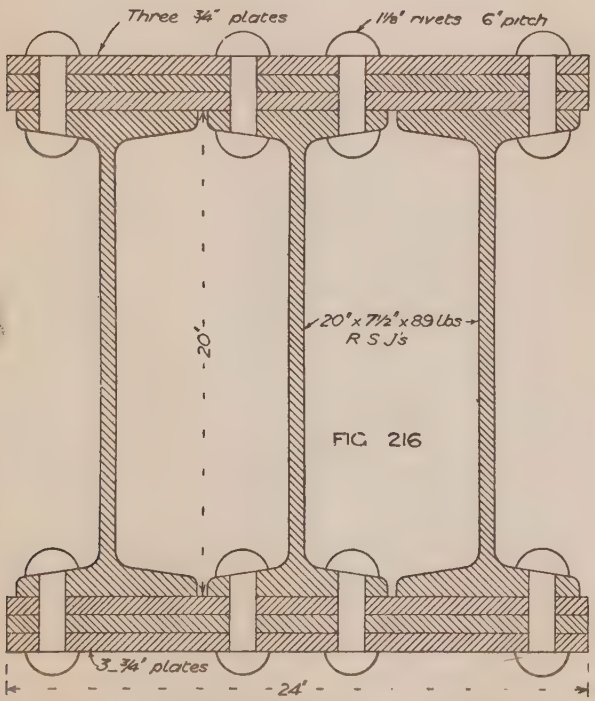


Fig. 216.—Cross-section of Rolled Joist Compound Girder, span 35 ft., to carry 100 tons.

will take the section given in Fig. 216, and find the safe distributed load and the remaining particulars of construction. The clear span is 35 ft., and the total length 40 ft.

The weight of the girder will be made up as follows:

Rolled joists and inner plates $(26{\cdot}2 \times 3 + 24 \times 1\frac{1}{2}) \times 3\frac{1}{3} = 382$ lbs. per ft. run

$382 \times 40 =$ . . . . . 15,280 lbs.

Second plates, approximate length (see Designing Ironwork, 2nd Series, Part I.) $40 \times {\cdot}85 = 34$ ft.

Third plates (see ditto) $40 \times {\cdot}65 = 26$ ft.

$2\,(34 + 26) \times 2 = 240$ ft. sup.

240×30 lbs. per ft. sup. = ... 7,200 lbs.

22,480 lbs.

Rivets, say $1\frac{1}{2}$ per cent. = ... 336 lbs.

22,816 lbs.

$\frac{22{,}816}{2{,}240}$ = say, 10 tons total weight.

Effective span, centre to centre of bearing surfaces,

$\frac{40 + 35}{2} =$ . . . . . 37·5 ft.

Effective depth, say joist flanges equal to $\frac{3}{4}$-in. plate continuous, then between centres of gravity of flanges = ... $21\frac{1}{2}$ in.

$$\text{Stress in flanges} = \frac{WL}{8\,D} = \frac{(W + 10)\,37{\cdot}5}{8 \times 1{\cdot}8}$$

$$\text{Safe working stress} = \frac{\text{ult. tension} \times \text{depth ft.}}{0{\cdot}3 \text{ span ft.}} =$$

$\frac{30 \times 1{\cdot}8}{0{\cdot}3 \times 37{\cdot}5} =$ . . . . . 4·8 tons

$$\text{Stress} = \text{Resistance}$$

$$\frac{(W + 10)\,37{\cdot}5}{8 \times 1{\cdot}8} = 4{\cdot}8 \times 4 \times \tfrac{3}{4}\,(24 - 4 \times 1\tfrac{1}{8})$$

$$\therefore W = \frac{4{\cdot}8 \times 4 \times \frac{3}{4}\,(24 - 4 \times 1\frac{1}{8})\,8 \times 1{\cdot}8}{37{\cdot}5} - 10$$

$$= \frac{14{\cdot}4 \times 19{\cdot}5 \times 14{\cdot}4}{37{\cdot}5} - 10$$

$$= \frac{4043{\cdot}5}{37{\cdot}5} - 10 = 97{\cdot}83, \text{ say 100 tons safe load.}$$

Bearing Surfaces—

Total load, say 110 tons, total bearing surface $2\ (40 - 35) =$ . . . . . 10 sq. ft.

Load per ft. sup. on bearing, $\frac{110}{10} =$ . . . 11 tons

which would be quite safe upon an ordinary York stone template 6 ins. thick.

Shear Strength of Webs—

Sectional area of webs $18 \times \cdot 6 \times 3 = 32 \cdot 4$

$\frac{\frac{1}{2} \times 110}{32 \cdot 4} = 1 \cdot 7$ tons per sq. in., which is well on the safe side.

Pitch of Rivets—

Rivets when cold would be $1\frac{1}{16}$ ins. diameter, making $1\frac{1}{8}$ ins. finished size.

$\frac{\text{Max. shear tons}}{\text{Depth web ft.}} = \frac{55}{1 \cdot 5} = 36 \cdot 7$ tons horizontal shear per ft. run, safe shear, say 5 tons per sq. in., $1\frac{1}{8}$-in. rivet, say 1 sq. in. area = 5 tons shear value, $\frac{36 \cdot 7}{5} = 7 \cdot 34$ rivets per ft. run between joists and flange.

There are 4 rivets in the cross-section, $\frac{7 \cdot 34}{4} =$ say, 2 rivets per ft. run = 6 ins. pitch. Strictly it will not require the pitch so close as this, owing to the extra depth given by the solid flanges. So that this girder will be quite safe under a distributed load of 100 tons.

A nearly similar section is given by Messrs. Dorman, Long & Co., but with $\frac{5}{8}$-in. plates instead of $\frac{3}{4}$ in., $\frac{7}{8}$-in. rivets instead of $1\frac{1}{8}$ ins., and 4 ins. pitch instead of 6 ins. The modifications bring the safe load for a clear span of 36 ft. according to their tables up to 164 tons, the ultimate strength of the steel being taken at 32 tons per sq. in.

Built up girders, when used for the main girders of bridges, are often of comparatively deep section, the stresses are thereby reduced, and a lighter girder results. The section, Fig. 216A, shows a typical arrangement of the main girder for a road or railway bridge of 60-ft. span, suitable for cross-girders either

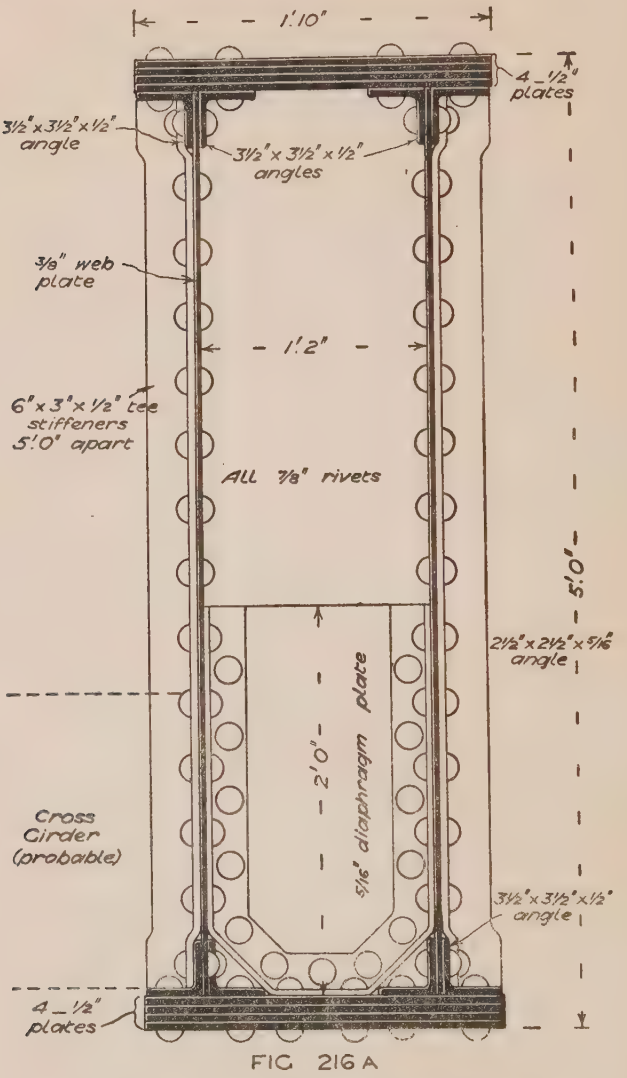

1'-10"
4 - 1/2" plates
3 1/2" x 3 1/2" x 1/2" angle
3 1/2" x 3 1/2" x 1/2" angles
3/8" web plate
1'-2"
6" x 3" x 1/2" tee stiffeners 5'-0" apart
All 7/8" rivets
5'-0"
2 1/2" x 2 1/2" x 5/16" angle
2'-0"
5/16" diaphragm plate
Cross Girder (probable)
3 1/2" x 3 1/2" x 1/2" angle
4 - 1/2" plates
FIG 216 A

FIG. 216A.—Cross-section of built-up Girder for 60-ft. span.

attached to the side above the bottom flange, or hung from below the bottom flange by bolts. Owing to the great proportionate depth and little width there would be considerable tendency to distortion if it were not for the diaphragm-plate inserted

between the webs behind each of the stiffeners. When the cross-girders are carried at the side there is only a narrow width of flange to rest upon, and they have to be securely fixed endways. At these points the diaphragm-plates have an important office to perform in transmitting the stress produced by the deflection of the cross-girders. When the diaphragms are too high to step over readily it is desirable to put a manhole through near the bottom, or instead of a round or oval manhole, which is expensive to cut, two diaphragm-plates vertically over each other may be arranged with sufficient space between to enable a man to pass through.

The question whether the stiffeners should be joggled as shown, or used straight with a packing-piece at the back, will depend upon the relative cost of the packing-piece as against the two joggles. Generally, in a girder over 3 ft. deep, the joggles will be cheaper than a packing-piece.

For buildings and other purposes, where the load is on top of the girder, it is necessary to keep the girder fairly shallow in depth, and the extra weight of metal to meet the additional stress produced is paid for by the increased height available for other purposes, or by the reduction possible in the total height. It should be remembered that every shallow girder requires a reduction in the working intensity of stress allowed in order that the deflection may not be excessive. This can be effected either by using a special factor of safety which varies with the ratio of depth to span, or by directly limiting the working stress.

CHAPTER XIX

Designing a Steel Plate Box Girder to carry 100 tons—Detailed Calculations and Working Drawing

An example may now be taken of a steel box girder to carry 100 tons distributed over a clear span of 50 ft. The calculations should be systematically entered upon a sheet of foolscap, or in a draughtsman's foolscap "record book," headed with the name of the contract for which the girders are being designed, and tabulated as follows:

Steel Plate Box Girder.

Clear span . . . . . 50 ft.
External dead load uniformly distributed . . . 100 tons.
Mean depth of girder, say $\frac{1}{20}$th span . . . 2·5 ft.
Width of flanges (least width between webs 12 ins.), say $\frac{1}{20}$th span . . . . . 2·5 ft.
Approximate weight of girder:

$$\frac{WL}{400} \times \sqrt{\frac{L}{d}} = \frac{100 \times 50}{400} \times \sqrt{\frac{50}{30}} = 12{\cdot}5 \times 1{\cdot}291 = 16{\cdot}1375, \text{ say} \quad . \quad . \quad . \quad . \quad 16 \text{ tons.}$$

Gross load 100 + 16 = ... 116 tons.
Safe load per sq. ft. on York stone bed . . . 15 tons.
Chamfer on edge . . . . . 3 ins.
Length of each bearing surface:

$$\frac{\frac{1}{2}\ (100 \text{ tons load} + 16 \text{ tons girder})}{2{\cdot}5 \text{ ft. width flange} \times 15 \text{ tons safe load}} = 1{\cdot}55 \text{ ft.}$$

say . . . . . 18 ins.
Total length of girder 50 + 2 (·25 + 1·5) = . . 53·5 ft.
Effective span centre to centre of bearings 53·5 − 1·5 = ... 52 ft.
Stress in flanges at centre $\frac{WL}{8\,D} = \frac{116 \times 52}{8 \times 2{\cdot}5} = 301{\cdot}6$, say . . . . . 300 tons.

Factor of safety to limit deflection :

$\frac{\text{span}}{\text{depth}} \times 0{\cdot}3 =$ 6

Ultimate stress per sq. in. . . . . 30 tons.

Working stress per sq. in. $\frac{30}{6}$ . . . . . 5 tons.

Sectional area of each flange $\frac{300}{5} =$ . . . . 60 sq. ins.

Angle bars, say 4 ins. × 4 ins. × $\frac{5}{8}$ in.

Approximate thickness of flanges at centre =

$\frac{60 \text{ sq. ins.}}{30 \text{ ins. wide}} =$ . . . . . 2 ins.

Approximate depth of web between rivet holes, 2·5 ft. —2 ins. for flanges—4 ins. for angles = . . 2 ft.

Maximum shear = reaction at support = $\frac{116}{2} =$. 58 tons.

Maximum shear stress per foot in depth and per foot run horizontal $\frac{58}{2} =$ . . . . . 29 tons.

Diameter of rivets, say . . . . . 1 in.

Effective depth of web = total depth—2 rivet holes = 30 ins. mean depth of girder—2 ins. for flange —(2 × 1) for rivets . . . . . 26 ins.

Thickness of web plates, allowing 3 tons per sq. in. safe shear stress,

$\frac{58 \text{ tons}}{3 \text{ tons per sq. in.} \times 26 \text{ ins. width} \times 2 \text{ webs}} =$

·371, say . . . . . $\frac{3}{8}$ in.

Sectional area of 1 rivet . . . . . ·8 sq. in.

Safe shear stress per sq. in. rivet area, single shear . 4·5 tons.

Double shear, 4·5 × $1\frac{1}{2} =$ . . . . . 6·75 tons.

Safe shear stress per rivet double shear 6·75 × ·8 . 5·4 tons.

Number of rivets per ft. run, double shear $\frac{29}{5{\cdot}4} = 5{\cdot}37$,

say . . . . . 6 rivets

Pitch of rivets $\frac{2 \text{ webs} \times 12 \text{ ins.}}{6 \text{ rivets}} =$ . . . . 4 ins.

Bearing pressure of rivets per sq. in. not to exceed . 10 tons.

Actual pressure in holes of webs per sq. in.

$$\frac{\text{29 tons stress per ft.}}{\text{6 rivets} \times \text{bearing area per rivet } (1 \times \cdot 375)} = 12{\cdot}8 \text{ tons.}$$

As this exceeds the limit the pitch must be reduced

at ends to $4 \times \frac{10}{12{\cdot}8} = 3{\cdot}125$, say . . . 3 ins.

But the 4-in. pitch may be retained if desired for

$\frac{10}{12{\cdot}8} \times \frac{50}{2} = 19{\cdot}5$ ft. each side of centre.

Equivalent plate thickness of one side of four angles

$\frac{4\,(4 \times \frac{5}{8})}{30\text{-in. flange}} =$ . . . . . $\frac{1}{3}$ in.

Least thickness of plates in top flange, rivets in compression not deducted $2 - \frac{1}{3} =$ . . . . $1\frac{2}{3}$ ins.

Plates in top flange, say three $\frac{5}{8}$-in. plates . . $1\frac{7}{8}$ in.

Least thickness of bottom flange,

Net section required = ... 60 sq. ins.

Net section of four angle sides less four 1-in. rivets $= 4\,(4 - 1) \times \frac{5}{8} =$ . . . . 7·5 sq. ins.

Net section of plates $60 - 7{\cdot}5 =$ . . . 52·5 sq. ins.

Net width of plates $30 - 4 \times 1$-in. holes = . 26 ins.

Least thickness of flange $= \frac{52{\cdot}5}{26} = 2{\cdot}02$, say . 2 ins.

Plates in bottom flange, say one $\frac{3}{4}$-in. and two $\frac{5}{8}$-in. plates = ... 2 ins.

Steel plates may now be obtained in one piece at ordinary prices, up to 20 cwts., 30 ft. long, $2\frac{1}{2}$ ins. wide, $1\frac{1}{2}$ ins. thick, 60 ft. super, and at extra cost up to 2 tons, 60 ft. long, and 120 ft. super, but only one of these limits may be reached in the same plate.

In this case it will probably be well to put a joint in the inner flange plates and the web plates. The angles may be in one length.

Cover to joint in $\frac{3}{4}$-in. bottom flange plate.

Net sectional area of part cut through $\cdot 75\,(30 - 4 \times 1)$ 19·5 sq. ins.

Sectional area of 1 rivet, say = ... ·8 sq. in.

No. of rivets each side of joint $\frac{19{\cdot}5}{\cdot 8} = 24{\cdot}375$, say . 24 rivets.

Length of cover plate $\frac{24 \text{ rivets}}{4 \text{ in a row}} \times \frac{4 \text{ ins. pitch} \times \text{two ends of cover}}{12 \text{ ins. in 1 ft.}}$ . . . 4 ft. long.

Cover to joint in $\frac{5}{8}$-in. top flange plate, being in compression with planed butt joints need only be small, say 12 rivets in two rows on each side = 16 ins. long.

Cover to web plates on insides only, between angles behind stiffeners:

Thickness . . . . . $\frac{3}{8}$ in.
Width . . . . . 6 ins.
Rivets $\frac{3}{4}$ in., pitch 4 ins.

Stiffeners, between angles only, on outside:

At ends $3\frac{1}{2}$ ins. $\times$ $3\frac{1}{2}$ ins. by $\frac{1}{2}$ in. angles.
Over edge of bearing surfaces and 4 ft. to 6 ft. apart between, say, 6-in. $\times$ 3-in. $\times$ $\frac{1}{2}$-in. tees.

End plates to close girder, $\frac{3}{8}$ in. thick, bolted through end stiffener angles by eight $\frac{3}{4}$-in. bolts and nuts.

Desirable limit for deflection $50 \times \frac{l}{480} \times 12 =$. $1\frac{1}{4}$ in.

Calculated deflection:

S = stress allowed, tons per sq. in.
L = span in inches.
E = modulus of elasticity in tons.
D = mean depth in inches.
d = deflection in centre in inches.

$$d = \frac{SL^2}{4\,ED} = \frac{5\,(50 \times 12)^2}{4 \times 13{,}000 \times 30} = \quad . \quad . \quad . \quad 1{\cdot}154 \text{ in.}$$

and this would have been still more if the flange stress had not been reduced below the usual $6\frac{1}{2}$ tons per sq. in.

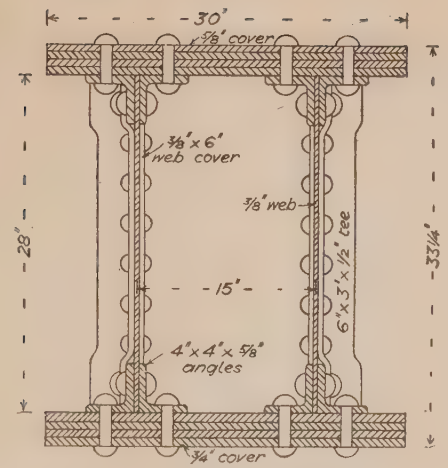
Permanent camber $\frac{1}{8}$ in. to $\frac{1}{4}$ in. per 10-ft. span, say . 1 in.
Building camber = 1·154 + 1 = 2·154, say . . $2\frac{1}{4}$ ins.

The design may now be proceeded with, first finding the proper length of flange-plates by drawing a parabola for each flange, of a span equal to the centre distance of bearings and a height equal to total thickness, including equivalent value of angles, to any convenient scales, the height being to a larger

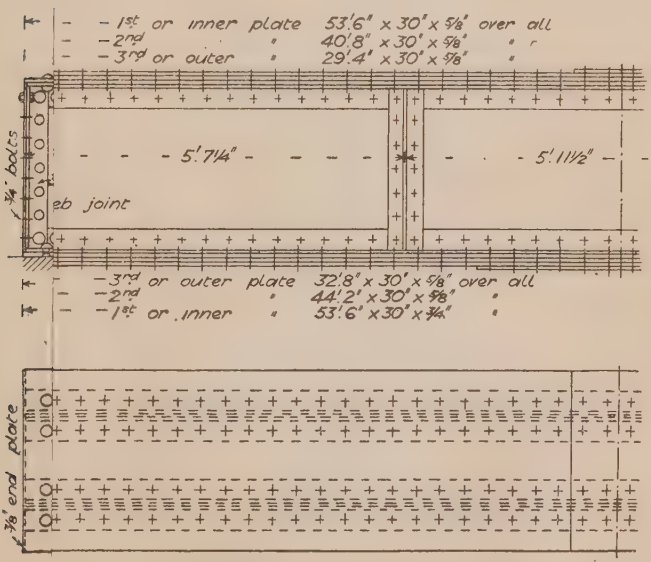
scale than the length. This will be seen upon the upper part of the diagram. Where the parabola cuts the level of the inner edge of each plate, a distance equal to half-cover, according to thickness, must be added to obtain the proper termination of the plate. Then the section, elevation, and plan may be drawn in as shown to complete the design, and the actual weight may then be estimated. Instead of reducing the pitch of the rivets towards the ends of the girder, a sufficient bearing area in the holes may be obtained by increasing the thickness of the web-plates, making them from each end 4 ft. $6\frac{1}{4}$ ins. by $\frac{1}{2}$ in., 3ft. $9\frac{1}{4}$ ins. by $\frac{7}{16}$ in., and the remainder $\frac{3}{8}$ in. thick.

The safe shear stress has been taken in this case at 4·5 tons and 6·75 tons per sq. in., instead of 5 tons and 7·5 tons as in the previous chapters; the latter figures are safe, and would have permitted the pitch to be increased to 3·75 ins. and 4·8 ins. at the ends and centre respectively, but the stress is often limited to the first-named figures.

SD TONS



SECTION ON CENTRE LINE OF GIRDER


1st or inner plate 53'6" x 30" x 5/8" over all
2nd " 40'8" x 30" x 5/8" "
3rd or outer " 29'4" x 30" x 5/8" "
3/4" bolts
5'7 1/4"
5'11 1/2"
web joint
3rd or outer plate 32'8" x 30" x 5/8" over all
2nd " 44'2" x 30" x 5/8" "
1st or inner " 53'6" x 30" x 3/4" "

[*To face p.* 154.

CHAPTER XX

Steel Roof-Trusses for various Spans.—Designing a North-Light Roof

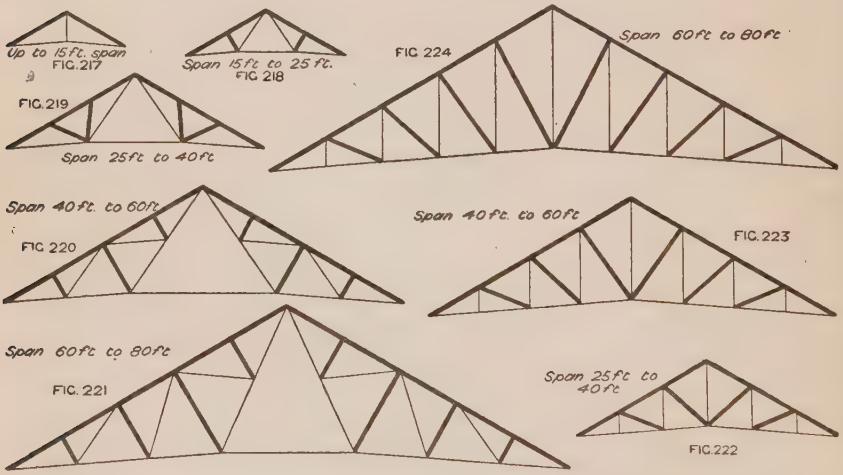
Steel roof-trusses are now universal over factories and workshops, and are very largely used over other buildings. The principal difference from wooden trusses is that the tie-rods are cambered, and king and queen post trusses give way to trussed rafter forms. Many manufacturers keep the smaller span trusses in stock in two qualities, light for galvanised-iron roofing and heavy, for slating. The use of flat bars with joint-plates and rivets usually results in a cheaper truss than when round rods and single pins or bolts are used.

For galvanised-iron covering the trusses are generally curved, which saves the formation of a ridge; but these curved roofs are always unsightly. In designing roofs to cover a considerable area the question often arises whether it is more economical to adopt numerous trusses of small span or a smaller number of trusses of greater span. It will be found that when the number of separate roofs is reduced and the trusses increased in span up to a maximum of about 60 ft. there will be a saving in cost owing to the more accurate proportioning of the material to the stress, and frequently also there will be an additional saving in the supports.

When the span and type of truss are determined, the next point to consider is the number of bays in each rafter. This will fix the number of purlins and the span of the common rafters. In straight rafter roofs the number of bays formed by the trussing on each side will vary with the span as follows:

1	bay	up	to	15	ft.	span.
2	bays	from	15	ft.	to 25	ft. span.
3	„		25	ft.	to 40	ft. „
4	„		40	ft.	to 60	ft. „
5	„		60	ft.	to 80	ft. „

The ordinary types for steel trusses of moderate span are shown in the diagram. Figs. 217 to 221 are favourite forms, and are known as "trussed rafter roofs." Figs. 222 to 224 are on the king and queen post principle, and make good, workmanlike arrangements. In every case the purlins are supposed to come over the points of support, that is, where the struts meet the principal rafter. There are two objectionable terms in connection with roofs that should be avoided: they are the word "principal" instead of truss, and "bearing" instead of span. There are



FIGS. 217 to 224.—Types of Steel Trusses for various spans.

many other forms of symmetrical bracing, especially in roofs over 50 ft. span, the chief variation being the carrying of the end of truss down the wall and curving the soffit in order to improve the appearance and produce less overturning effort.

A truss similar to Fig. 220 will be found worked out in detail by the present author in "The Practical Designing of Structural Ironwork" (Spon; 8s. 6d.).

We will now proceed to calculate and design a truss for a north-light roof. This form called also a "saw-tooth" or "weaving shed" roof, is used where a good, unvarying light is needed, as in weaving-sheds, show-rooms, printing works, etc.,; but, of course, the possibillity of using it depends to some extent

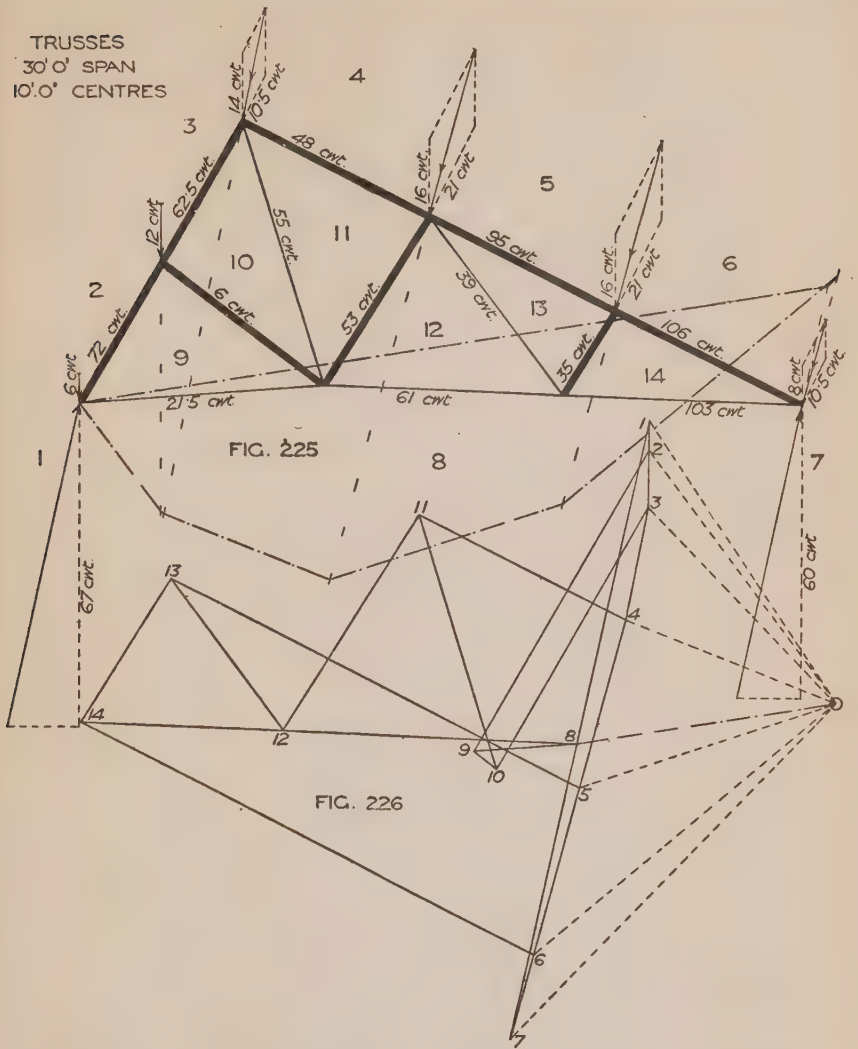


FIG. 225.—Frame diagram of North-Light Roof Truss with wind on the long side.
FIG. 226.—Stress diagram for Fig. 225.

upon the aspect of the building. Such trusses are usually from 20 ft. to 30 ft. span, and they should be not wider than twice the height from floor to underside of truss, or the light will not be sufficiently distributed. The trusses may be carried by a wall or

girder, the supporting columns for the latter being, say, below every third truss. The angle of the steep or glazed side of roof will be 60 deg., and the flat side, say 26½ deg., which is the flattest desirable slope for slate coverings. The wind may com

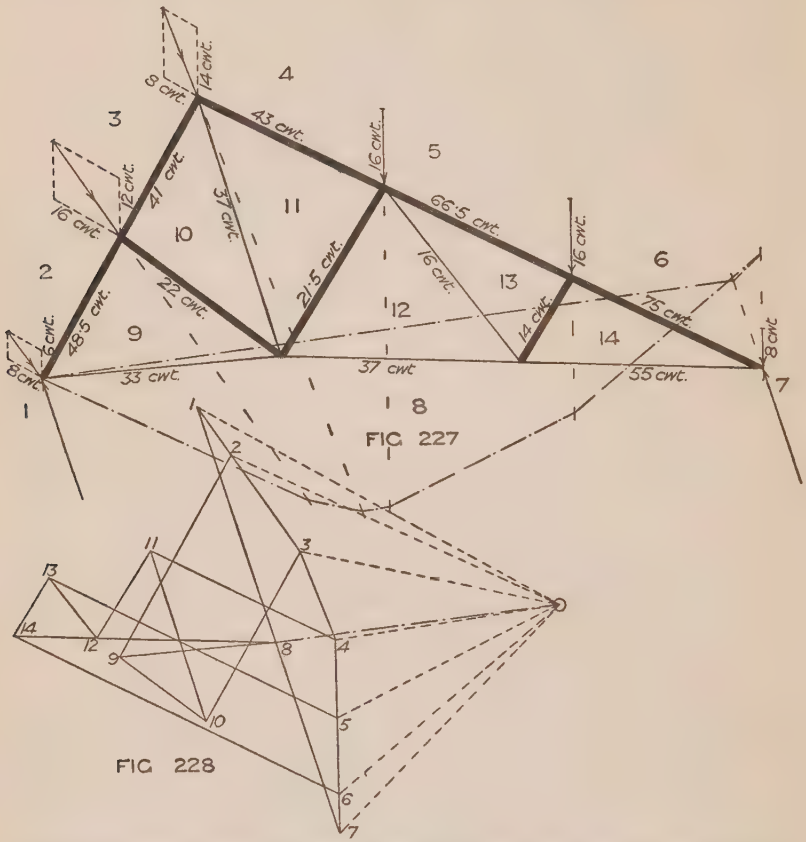


FIG 227

FIG 228

Fig. 227.—Frame diagram of North-Light Roof Truss with wind on the short side.
Fig. 228.—Stress diagram for Fig. 227.

from either side, and, as the truss is not symmetrical, separate stress diagrams will be required for each case. The dead load may be taken at 21 lbs. per ft. super. and the wind at 28 lbs. normal to slope of roof. To set off the bracing, the rafters are divided into equal parts, and also the horizontal line joining the supports. With trusses 30 ft. span, 10 ft. centre to centre loaded

as above, and wind on the flatter side, the frame diagram will be as Fig. 225, and the stress diagram as Fig. 226. With the wind on the steep side the frame diagram will be as Fig. 227, and the stress diagram as Fig. 228. A roof of this character will be found in the catalogue of some of the structural engineeers; for example, Fig. 229 is taken from the catalogue of Messrs. A. & J. Main & Co., Ltd., of Glasgow and London. The calculations for the different members will be as follows, care being taken to allow for the maximum stress from either diagram.

The maximum stress in the principal rafter on the long side is

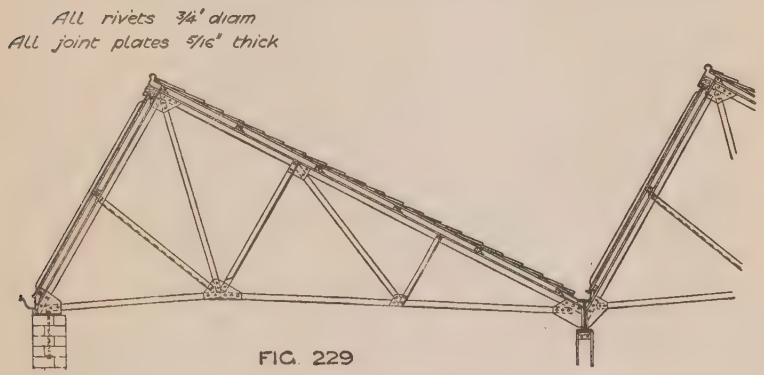


FIG. 229.—Elevation of North-Light Roof Truss made by Messrs. A. and J. Main and Co., Ltd.

106 cwts., as shown in Fig. 225. Try a $3\frac{1}{2}$-in. by $3\frac{1}{2}$-in. by $\frac{7}{16}$-in. tee, then $W = \dfrac{fS}{1 + \dfrac{m}{nq}\left(\dfrac{l}{d}\right)^2} = \dfrac{6{\cdot}5 \times 2{\cdot}9}{1 + \dfrac{2{\cdot}5}{\frac{1}{2} \times 3{,}750}\left(\dfrac{104}{3{\cdot}3}\right)^2} = 8{\cdot}1$ tons, which is more than enough, but a smaller size would not be sufficient. It may, however, happen that the truss will be used to carry purlins, as in Fig. 229, giving a distribution of loads as in Fig. 230. Bending moments will then be introduced in addition to the direct stresses, and it would be wise to make provision for them. Then the maximum stress in the principal rafter = 106 cwts., or 5·3 tons, the maximum bending moment from purlins on the same bay $= \dfrac{Wab}{L} = \dfrac{9{\cdot}6 \times 3{\cdot}46 \times 5{\cdot}2}{8{\cdot}6} =$

20 cwt.-ft. $= \frac{20 \times 12}{20} = 12$ ton-ins., and the unsupported length of rafter $= 8$ ft. 8 ins. $= 104$ ins. Try a 4-in. by 5-in. by $\frac{1}{2}$-in. tee section, then the moment of inertia I given by the tables $= 10{\cdot}46$, $x = \frac{5^2 \times \frac{1}{2} + \frac{1}{2}^2 (4 - \frac{1}{2})}{2 \times 4{\cdot}25} = 1{\cdot}58$ ins., and $y = 5 - 1{\cdot}58 = 3{\cdot}42$ ins., $Z_x = \frac{I}{x} = \frac{10{\cdot}46}{1{\cdot}58} = 6{\cdot}62$, and $Z_y = \frac{I}{y} = \frac{10{\cdot}46}{3{\cdot}42} = 3{\cdot}06$. Then $\frac{W}{A} + \frac{M}{Z_x} = \frac{5{\cdot}3}{4{\cdot}25} - \frac{12}{6{\cdot}62} = 1{\cdot}25 + 1{\cdot}81 = 3{\cdot}06$ tons per sq. in. compression, and $\frac{W}{A} - \frac{M}{Z_y} = \frac{5{\cdot}3}{4{\cdot}25} - \frac{12}{3{\cdot}06} = 1{\cdot}25 - 4 =$

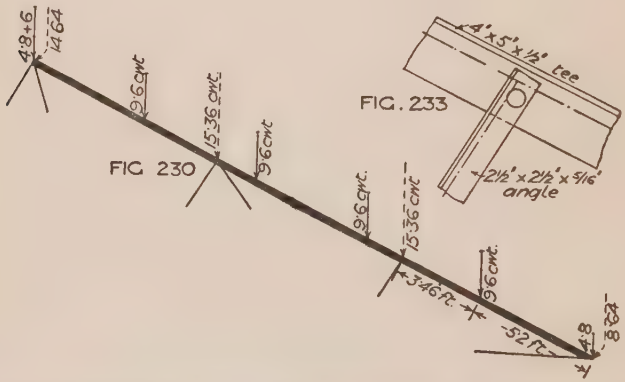


FIG. 230.—Diagram showing loads on Joints calculated from positions of Purlins.
FIG. 233.—Detail of Joint at head of second Strut.

2·75 tons per sq. in. tension. By Gordon's formula the maximum allowable compressive stress will be

$$W = \frac{f}{1 + \frac{m}{nq}\left(\frac{l}{d}\right)^2} = \frac{6\frac{1}{2}}{1 + \frac{2{\cdot}5}{\frac{1}{2} \times 3{,}750}\left(\frac{104}{3{\cdot}8}\right)^2} = 3{\cdot}25 \text{ tons per sq. in.}$$

For members, 2-9 and 3-10, with a maximum stress of 72 cwts. $= 3{\cdot}6$ tons, and unsupported length of 6 ft. 8 ins. $= 80$ ins., try a 3-in. by 3-in. by $\frac{3}{8}$-in. section, then

$$W = \frac{fS}{1 + \frac{m}{nq}\left(\frac{l}{d}\right)^2} = \frac{6{\cdot}5 \times 2{\cdot}11}{1 + \frac{2{\cdot}5}{\frac{1}{2} \times 3{,}750}\left(\frac{80}{2{\cdot}2}\right)^2} = 5{\cdot}04 \text{ tons,}$$

which allows a good margin. For member 11-12, with a maximum stress of 53 cwts. = 2·65 tons, and an unsupported length of 8 ft. 4 ins. = 100 ins., try a 3-in. by 3-in. by $\frac{3}{8}$-in. angle section, then

$$W = \frac{fS}{1 + \frac{m}{nq}\left(\frac{l}{d}\right)^2} = \frac{6{\cdot}5 \times 2{\cdot}11}{1 + \frac{2{\cdot}5}{\frac{1}{2} \times 3{,}750}\left(\frac{100}{2{\cdot}2}\right)^2} = 3{\cdot}71 \text{ tons},$$

which allows a good margin. For member 9-10, with a maximum stress of 22 cwts. = 1·1 tons, and an unsupported length of 8 ft. 4 ins. = 100 ins., try a $2\frac{1}{2}$-in. by $2\frac{1}{2}$-in. by $\frac{5}{16}$-in. angle, then

$$W = \frac{fS}{1 + \frac{m}{nq}\left(\frac{l}{d}\right)^2} = \frac{6{\cdot}5 \times 1{\cdot}47}{1 + \frac{2{\cdot}5}{\frac{1}{2} \times 3{,}750}\left(\frac{100}{1{\cdot}8}\right)^2} = 1{\cdot}9 \text{ tons}.$$

For member 13-14, with a maximum stress of 35 cwts. = 1·75 tons, and an unsupported length of 4 ft. 2 ins. = 50 ins., try a 2-in. by 2-in. by $\frac{1}{4}$-in. angle, then

$$W = \frac{fS}{1 + \frac{m}{nq}\left(\frac{l}{d}\right)^2} = \frac{6{\cdot}5 \times 0{\cdot}94}{1 + \frac{2{\cdot}5}{\frac{1}{2} \times 3{,}750}\left(\frac{50}{1{\cdot}4}\right)^2} = 2{\cdot}25 \text{ tons},$$

which allows a small margin, but where $\frac{3}{4}$-in. rivets are used the size should be not less than $2\frac{1}{2}$ ins. by $2\frac{1}{2}$ ins. by $\frac{5}{16}$ in. Members 8-12 and 8-14 have a maximum tensile stress of 103 cwts. = 5·15 tons, and allowing $6\frac{1}{2}$ tons per sq. in., the net sectional area required $= \frac{5{\cdot}15}{6{\cdot}5} = 0{\cdot}8$ sq. in., which will be given by a 3-in. by $\frac{3}{8}$-in. flat bar, deducting one $\frac{3}{4}$-in. rivet hole. Member 8-9, with a maximum stress of 33 cwts. = 1·65 tons, will require a net area of $\frac{1{\cdot}65}{6{\cdot}5} = 0{\cdot}25$ sq. in., which will be given by a 2-in. by $\frac{1}{4}$-in. flat bar, and the same section would be just sufficient for member 12-13, but they should be made $2\frac{1}{2}$ ins. by $\frac{5}{16}$ in. Member 10-11, with a maximum stress of 55 cwts. = 2·75 tons, requires an area of $\frac{2{\cdot}75}{6{\cdot}5} = 0{\cdot}42$ sq. in., which will be given by a $2\frac{1}{2}$-in. by $\frac{5}{16}$-in. flat bar.

The 30 ft. span girders carrying the ends of the trusses will each have to support a total concentrated load at 10 ft. from each

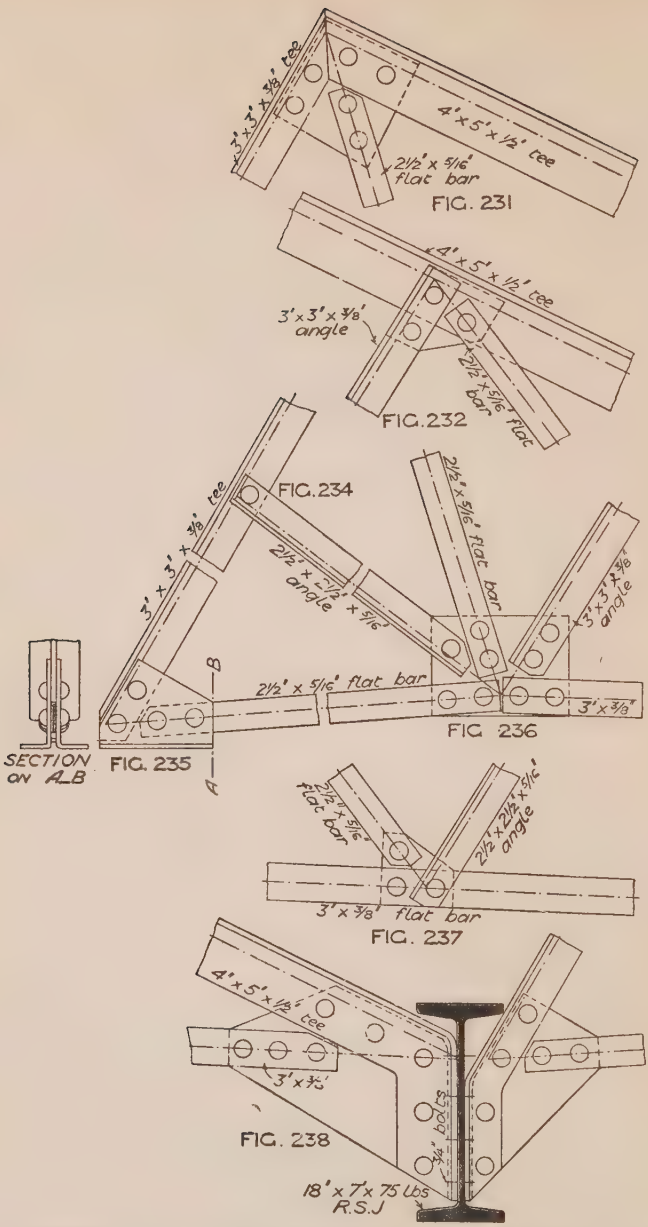

3" x 3" x 3/8" tee
4" x 5" x 1/2" tee
2 1/2" x 5/16" flat bar
FIG. 231
4" x 5" x 1/2" tee
3" x 3" x 3/8" angle
2 1/2" x 5/16" flat bar
FIG. 232
FIG. 234
3" x 3" x 3/8" tee
2 1/2" x 2 1/2" x 5/16" angle
2 1/2" x 5/16" flat bar
3" x 3" x 3/8" angle
B
2 1/2" x 5/16" flat bar
3" x 3/8"
FIG 236
SECTION ON A.B
FIG. 235
A
2 1/2" x 5/16" flat bar
2 1/2" x 2 1/2" x 5/16" angle
3" x 3/8" flat bar
FIG. 237
4" x 5" x 1/2" tee
3" x 3/8"
3/4" bolts
FIG. 238
18" x 7" x 75 lbs R.S.J

FIG. 231.—Detail of Joint at junction between principal Rafters. FIG. 232.—Detail of Joint at head of main Strut of long Rafter. FIG. 234.—Detail of Joint between Strut and short Rafter. FIG. 235.—Detail of Joint at left-hand Support. FIG. 236.—Detail of Joint at junction of Tie-bars. FIG. 237.—Detail of Joint between secondary Strut and Tie-bar. FIG. 238.—Detail of Joint at right-hand Support.

end equal to the vertical components of the reactions from the two trusses = 67 + 60 = 127 cwt., or 6·35 tons. Then the maximum bending moment will be the reaction at either end multiplied by the distance to the first load = 6·35 tons × 10 ft. = 63·5 ton-ft., and the tabular value = 63·5 × 8 = 508, requiring (B.S.B.28) 18-in. by 7-in. by 75-lbs. rolled steel joists.

The details of the joints may then be drawn out as shown in Figs. 231 to 238, the number of rivets being proportioned to the stress in the members, and the members so arranged that the lines down the centre of gravity of their cross-section shall intersect in the same point to avoid secondary stresses.

CHAPTER XXI

Steel-Framed Sheds—Failures from Ignorance in Designing Details of Stresses

During the last ten years there has been a large number of steel-framed sheds erected in all parts of the world, and many of them have failed from the same cause—viz., the absence of bracing to the stanchions, and the omission to secure the stanchions sufficiently at their lower ends. Recently several skating rinks have been erected with the same want of precaution, and it cannot be too strongly asserted that temporary buildings require as great a margin of safety as permanent structures. If the trusses are merely bolted down on the top of the stanchions, the latter will be in the condition of fixed at one end and nearly free at the other, but if the trusses are so arranged that they can be connected to the stanchions by diagonal braces, the resistance to bending will be increased very considerably. The extreme difference in this way may approach 16 to 1, so that there is a very cogent reason for bracing being adopted.

An example may be taken of a shed roof 50 ft. span, 10 ft. rise, camber 1 in. per foot, and 17 ft. 6 ins. from ground to eaves, the trusses 10 ft. centre to centre, covered with galvanised corrugated iron on angle purlins. For 50 ft. span the principal rafters may be divided into three bays, and choice may be made of either of the forms shown in Figs. 239 to 241. Fig. 241 will, perhaps, be the best to adopt. The load may be taken as 14 lbs. per foot super. of sloping surface for the structural load, and 28 lbs. per foot super. on one side normal to the surface for the wind. This will give the distribution of loading shown in the frame diagram, Fig. 242, the stanchion down to the junction with brace being taken as part of the truss. The reactions from the two sets of loads may be taken separately, the structural load being divided equally over the two supports, and the wind

loads produced to meet the springing line of the truss, and the proportion going to each support calculated according to the position of the intersections with this line. The reciprocal stress diagram will be as Fig. 243, from which the stresses may be scaled off, and the amounts inserted on the frame diagram. The calculation of the scantlings will then proceed as follows, taking the maximum for each member upon whichever side it occurs. For the principal rafter with a maximum stress of 226 cwts. = 11·3 tons, and an unsupported length of 9 ft., try a 4-in. by 4-in

FIG. 239

FIG. 240

FIG. 241

FIGS. 239 to 241.—Types of Steel Trusses for 50 ft. span.

by $\frac{1}{2}$-in. tee, then, allowing a factor of safety of 4, and making f = working stress per sq. in., and S = sectional area, the total safe load

$$W = \frac{fS}{1 + \frac{m}{nq}\left(\frac{l}{d}\right)^2} = \frac{6{\cdot}5 \times 3{\cdot}75}{1 + \frac{2{\cdot}5}{\frac{1}{2} \times 3{,}750}\left(\frac{108}{3{\cdot}8}\right)^2} = 11{\cdot}8 \text{ tons},$$

which is just sufficient. For members 13-14 and 16-17, with a maximum stress of 52 cwts. = 2·6 tons, and unsupported length of 5 ft. 6 ins., try a $2\frac{1}{2}$-in. by $2\frac{1}{2}$-in. by $\frac{5}{16}$-in. angle, then

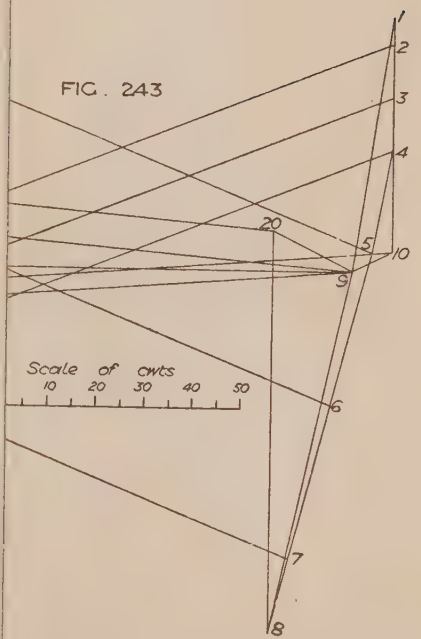
$$W = \frac{fS}{1 + \frac{m}{nq}\left(\frac{l}{d}\right)^2} = \frac{6{\cdot}5 \times 1{\cdot}47}{1 + \frac{2{\cdot}5}{\frac{1}{2} \times 3{,}750}\left(\frac{66}{1{\cdot}8}\right)^2} = 3{\cdot}4 \text{ tons},$$

so that this section will do, and the same size should also be used for members 11-12 and 18-19. Members 9-10 and 9-20 must be calculated for 18-cwt. tension, and 9 cwt. = 0·45 ton compression, with an unsupported length of 10 ft. 6 ins. Try a $2\frac{1}{2}$-in. by $2\frac{1}{2}$-in. by $\frac{5}{16}$-in. angle, assuming both ends pivoted, then

$$W = \frac{fS}{1 + \frac{m}{nq}\left(\frac{l}{d}\right)^2} = \frac{6{\cdot}5 \times 1{\cdot}47}{1 + \frac{4}{\frac{1}{2} \times 3{,}750}\left(\frac{126}{1{\cdot}8}\right)^2} = 0{\cdot}83 \text{ ton.}$$

This section will be ample for both tension and compression, but a smaller section should not be used. Allowing $6\frac{1}{2}$ tons per sq. in. tensile stress, members 10-11, 9-13, 9-17, and 20-19 with a maximum stress of 206 cwts. = 10·3 tons, will require an area of $\frac{10{\cdot}3}{6{\cdot}5} = 1{\cdot}58$ sq. ins., requiring two 3-in. by $\frac{3}{8}$-in. flat bars. Member 9-15, with a stress of 93·5 cwts. = 4·67 tons, requires an area of $\frac{4{\cdot}67}{6{\cdot}5} = 0{\cdot}72$ sq. in., which will be given by a 3-in. by $\frac{3}{8}$-in. flat bar. Members 12-13, 14-15, 15-16, and 17-18, with a maximum stress of 76 cwts. = 3·8 tons, require an area of $\frac{3{\cdot}8}{6{\cdot}5} = 0{\cdot}58$ sq. in., which will be given by a $2\frac{1}{2}$-in. by $\frac{3}{8}$-in. flat bar.

The purlins will have to carry a load of approximately 33 cwts. = 1·65 tons, over a span of 10 ft., producing a maximum bending moment of $\frac{33 \times 10}{8} = 41$ cwt.-ft., or $\frac{41 \times 12}{20} = 24{\cdot}6$ ton-ins. Try a 6-in. by 4-in. by $\frac{3}{8}$-in. angle, with the long side at right angles to the rafter, the distances of the neutral axis from the two edges being $x = 2$ ins., $y = 4$ ins., then the moment of inertia being 13·2, the section modulus on the compression side $= Z_x = \frac{13{\cdot}2}{2} = 6{\cdot}6$, and the section modulus on the tension side $= Z_y = \frac{13{\cdot}2}{4} = 3{\cdot}3$, and the stresses will be $\frac{M}{Z_x} = \frac{24{\cdot}6}{6{\cdot}6} = 3{\cdot}73$ tons per sq. in. compression in the table, and $\frac{M}{Z_y} = \frac{24{\cdot}6}{3{\cdot}3} = 7{\cdot}46$ tons per sq. in. tension in the leg, so that the size adopted will be just sufficient.



—Stress diagram for Fig. 242.

[*To face p.* 166.

The reactions may now be combined to find the resultants, and the resultants resolved into horizontal and vertical directions. The vertical portions will give direct compression on the stanchions, and the horizontal portions will be a part of the force producing bending moments. There will also be bending moments produced by the wind acting against the sides of the sheeting supported by the stanchions. As the stress produced thereby will be very considerable, it will be necessary to keep the allowance for wind pressure as low as possible. In 1887, in the discussion of a paper upon "The Stability of Chimney Shafts," read before the Society of Engineers, the writer gave his opinion that the pressure of the wind should be taken as varying with the height above ground level. He has since constructed an empirical formula, which, so far as experience goes, seems to give the minimum pressure that may be allowed for, in order to be within the limits of safety. The formula is $\log p = 1{\cdot}125 + 0{\cdot}32 \log h - 0{\cdot}12 \log w$, where $p =$ ultimate wind pressure in lbs. per sq. ft. necessary to be allowed for against a plane surface at right angles to the direction of the wind, $h =$ height of centre of gravity of surface considered above ground level in feet; $w =$ width in feet of part to be taken as one surface; and when the surface is inclined at θ degrees to the direction of the wind, the ultimate pressure normal to the surface may be taken as $p \sin \theta$, or its effect in the same direction as the wind as $p \sin^2 \theta$.

The vertical portion of the side between the foot of principal rafter and point of connection of brace and stanchion may be taken first. The height to centre of gravity will be 15·5 ft., and the width equal to the centre distance of stanchions 10 ft. Then $\log p = 1{\cdot}125 + 0{\cdot}32 \log 15{\cdot}5 - 0{\cdot}12 \log 10$, whence $p = 24{\cdot}32$, say 25 lbs. per sq. ft., and $\frac{25 \times 4 \times 10}{112} = 9$ cwts., which, added to the maximum horizontal reaction of roof 16·7 cwts., gives a total of 25·7 cwts., acting at level of foot of diagonal brace. Before the stresses can be worked any further the mode of bending which the stanchion will undergo must be considered, and it will be seen that the tendency is to move the upper part away from the centre line, as shown in Fig. 244. The horizontal

force of 25·7 cwts. will induce a corresponding opposite force at the ground level, and the stanchion between will be virtually as if composed of two cantilevers connected at their outer ends, and pressing against each other in opposite directions with the force of 25·7 cwts., the stress caused being equivalent to that of a load of 25·7 cwts. on the outer end of a cantilever $\frac{13{\cdot}5}{2} = 6{\cdot}75$ ft. span. The bending moment diagram for the stanchion under this loading

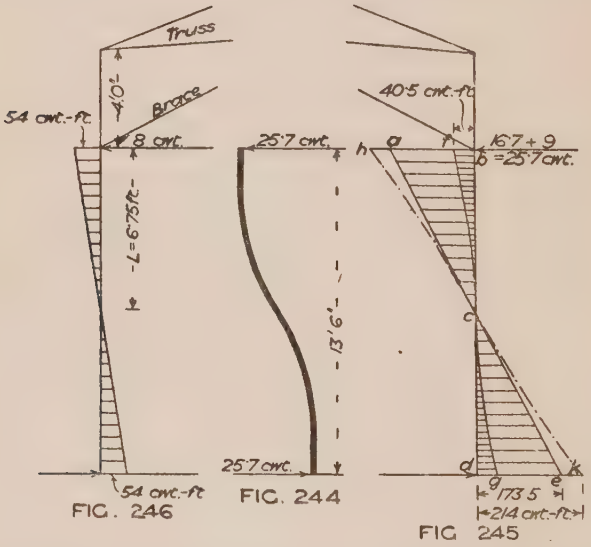


FIG. 246 FIG. 244 FIG 245

FIG. 244.—Line diagram on larger scale showing how the Stanchions tend to bend. FIG. 245.—Bending moments on Right-hand Stanchion. FIG. 246.—Bending moments on Left-hand Stanchion.

will be as shown by the two triangles *a b c*, *c d e*, in Fig. 245, where the maximum is $WL = 25{\cdot}7 \times 6{\cdot}75 = 173{\cdot}5$ cwt.-ft. The next step will be to ascertain the wind pressure on the sheeting supported by the stanchions below the level of junction with diagonal brace. The centre of gravity of this portion will be only 6·75 ft. above ground level, and the necessary pressure to be allowed for will be $\log p = 1{\cdot}125 + 0{\cdot}32 \log 6{\cdot}75 - 0{\cdot}12 \log 10$, whence $p = 18{\cdot}64$, say 20 lbs. per sq. ft. This will give a total distributed load on each of the virtual cantilevers of

$\frac{6{\cdot}75 \times 10 \times 20}{112} = 12$ cwts. Then $\frac{WL}{2} = \frac{12 \times 6{\cdot}75}{2} = 40{\cdot}5$ cwt.-ft. maximum bending moment. This will act in augmentation of the bending moments already found, but the load being distributed the ordinates will give parabolic outlines, as shown by *f b c*, *c d g*, in Fig. 245, and as these are overlapped by the previous diagram they can be added beyond the outline, as shown by *h b c* and *c d k*, giving a combined maximum at ground level of $173{\cdot}5 + 40{\cdot}5 = 214$ cwt.-ft. In addition to this there will be the axial load due to the vertical reaction of 74·5 cwts. to be considered when the size of the stanchion is being determined.

At the opposite end of the truss the stanchion will be under an axial load of 52·5 cwts., and a bending moment due to the wind on roof of $8 \times 6{\cdot}75 = 54$ cwt.-ft., as shown in Fig. 246; but as these are less than the results already taken into account for the stanchions they may be ignored. The details of this design will be considered in the next chapter, as the subject is of considerable importance, and involves questions of stability that have not been dealt with in any text-book.

CHAPTER XXII

Foundation for Steel-Framed Shed—Various Methods Adopted for Securing Stability

In continuation of the previous chapter upon the designing of a steel-framed shed, try a (B.S.B.16) 9-in. by 7-in. by 58-lb. rolled steel joist for the stanchions, which will then weigh $\frac{17{\cdot}5 \times 58}{112} = 9$ cwts. each, and have to resist a maximum dead load of $74{\cdot}5 + 9 = 83{\cdot}5$ cwts. $= 4{\cdot}17$ tons, and a maximum bending moment of 214 cwt.-ft $= \frac{214 \times 12}{20} = 128{\cdot}4$ ton-ins. This section will have an area of 17·06 sq. ins. and section modulus of 51, then $\frac{W}{A} \pm \frac{M}{Z} = \frac{4{\cdot}17}{17{\cdot}06} \pm \frac{128{\cdot}4}{51} = 0{\cdot}24 \pm 2{\cdot}52 = 2{\cdot}76$ tons per sq. in. compression and 2·28 tons per sq. in. tension. By Gordon's formula the allowable stress for bending in the weakest direction $= \frac{6{\cdot}5}{1 + \frac{1}{\frac{1}{2} \times 3{,}750}\left(\frac{17{\cdot}5 \times 12}{7}\right)^2} = 4{\cdot}3$ tons per sq. in. By London County Council table given on p. 83 of this series, the safe stress would be $6{\cdot}5 - \frac{1}{40}\left(\frac{l}{r}\right) = 6{\cdot}5 - \frac{1}{40} \times \frac{210}{1{\cdot}64}$ $= 3{\cdot}3$ tons per sq. in. It may be said that it is unfair to limit the safe load to that indicated by the least radius of gyration, but although the bending moment acts only in the direction of the greatest radius of gyration the dead load is not so limited, and incipient failure from this source would tend to occur in the weakest direction, making the final maximum stress to occur on one corner of a flange. We may, therefore, assume that the above method of working is correct where the stanchion is not prevented from bending sideways.

There are three methods by which stanchions can be securely

fixed at their base to resist the effect of leverage: (a) by gusseted flanges attached to floor-girders across the building; (b) by similar flanges bolted to concrete bases, or reinforced concrete bases, of large area; (c) by inserting the lower ends of the stanchions into deep piers of concrete sunk in the ground.

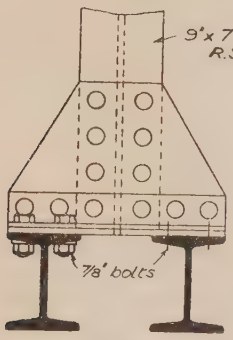


FIG. 247

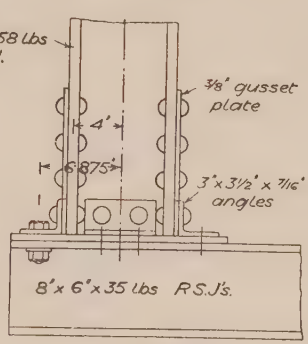


FIG 248

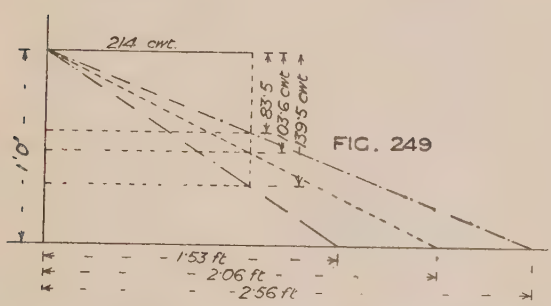


FIG. 247.—Front elevation of method of fixing Shed Stanchions to Floor-girders of iron Pier or Jetty. FIG. 248.—Side elevation of ditto. FIG. 249.—Parallelograms of Forces to find where Resultant cuts the ground line.

These methods may be independent of, or combined with, knee-bracing to the roof-trusses and to eaves-girders.

(a) This method would only be adopted when the shed is built on an iron pier or jetty, and the floor-girders can be used as part of the general framework. Such an arrangement would be that shown in Figs. 247 and 248. To calculate the rivets between the flange of stanchion and gusset-plate proceed as follows: The area of flange $= 7 \times 0{\cdot}924 = 6{\cdot}47$ sq. ins., then the total com-

pression to be resisted by shear $= 6{\cdot}47 \times 2{\cdot}76 = 17{\cdot}86$ tons, and allowing 2·2 tons for each $\frac{3}{4}$-in. rivet in single shear, the number required $= \frac{17{\cdot}86}{2{\cdot}2} = 8{\cdot}1$, say 8 rivets. The total tension to be resisted $= 6{\cdot}47 \times 2{\cdot}28 = 14{\cdot}75$ tons acting at the centre of gravity of the flange, say 4 ins. from centre line of stanchion. The centre line of the bolts will be 6·875 ins. from centre line of stanchion, so that the pull on the bolts $= \frac{14{\cdot}75 \times 4}{6{\cdot}875} = 8{\cdot}58$ tons; as there are four bolts, the stress on each will be $\frac{8{\cdot}58}{4} = 2{\cdot}15$ tons, and allowing 5 tons per sq. in. at bottom of thread, the area required will be $\frac{2{\cdot}15}{5} = 0{\cdot}43$ sq. in., which will be given by, say, $\frac{7}{8}$-in. bolts.

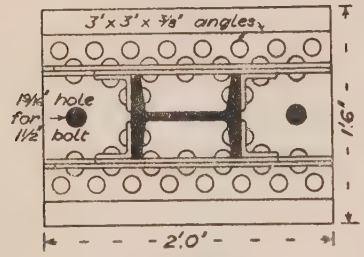


FIG. 250

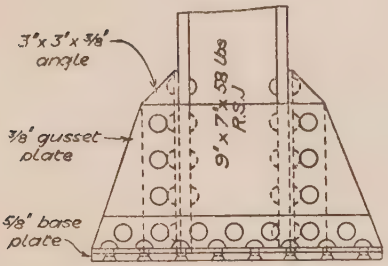


FIG. 251

FIG. 250.—Plan of base of Shed Stanchions.
FIG. 251.—Elevation of base of Shed Stanchions.

(b) A plain concrete base spread so as to take the load and bending moment without tilting would be worked out as follows: The dead load on stanchion is 83·5 cwts. and the bending moment 214 cwt.-ft. Take a point 1 ft. up, as in Fig. 249, then the horizontal force at this point to produce the bending moment $= 214$ cwts. Construct a parallelogram, of which this force gives the horizontal line and the direct load $74{\cdot}5 + 9 = 83{\cdot}5$ cwts. gives the vertical, then the diagonal will give the resultant, cutting the base at a point $\frac{214}{83{\cdot}5} = 2{\cdot}56$ ft. distant from centre line. The concrete may be assumed to be 1 : 3 : 6 mixture carrying a maximum safe load of 11 tons per sq. ft., then the size of

the base plate will be determined as follows. Assume the breadth to be 18 ins., then $S = \frac{W}{A} + \frac{M}{Z}$, or $11 = \frac{4{\cdot}17}{1{\cdot}5 \times d} + \frac{\frac{214}{20}}{\frac{1}{6} \times 1{\cdot}5 \times d^2}$, or $11 = \frac{2{\cdot}78}{d} + \frac{42{\cdot}8}{d^2}$, then $11 = \frac{2{\cdot}78d + 42{\cdot}8}{d^2}$, or $11d^2 = 2{\cdot}78d + 42{\cdot}8$, or $d^2 - {\cdot}25d = 3{\cdot}9$. Add to each side the square of half the co-efficient of d, then $d^2 - {\cdot}25d + \left(\frac{{\cdot}25}{2}\right)^2 = 3{\cdot}916$, take

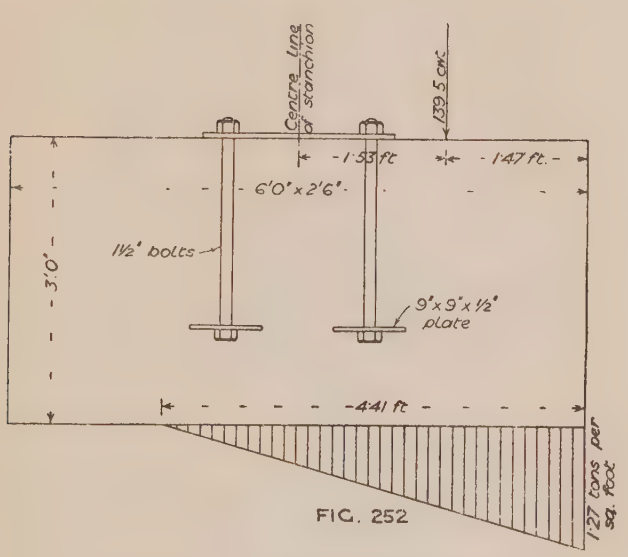


FIG. 252.—Plain Concrete Foundation for Stanchions.

the square root of each side, then $d - \left(\frac{{\cdot}25}{2}\right) = 1{\cdot}98$, whence $d = 1{\cdot}98 + {\cdot}125 = 2{\cdot}105$, say 2 ft. This will be too great a length to be obtained in the ordinary way of putting gusset-plates and angles parallel with the flanges of the stanchion; a more convenient method will be to arrange the base as shown in Figs. 250 and 251. The number of rivets to resist the compression by shear will be the same as in case (a), but for resisting the tension, the distance from centre of bolt to centre line of stanchion is increased to 9 ins. Therefore the total tension on

the single bolt will be $\frac{14{\cdot}75 \times 4}{9} = 6{\cdot}55$ tons, which at 5 tons per sq. in. will require an area at bottom of thread of $\frac{6{\cdot}55}{5} = 1{\cdot}31$ sq. in., which will be given by a $1\frac{1}{2}$-in. bolt. For the plain concrete base assume a size, say 6 ft. by 2 ft. 6 in. by 3 ft. deep, as in Fig. 252, weighing $6 \times 2{\cdot}5 \times 3 \times \frac{140}{112} = 56$ cwts. This weight must be added to the dead load and combined with the

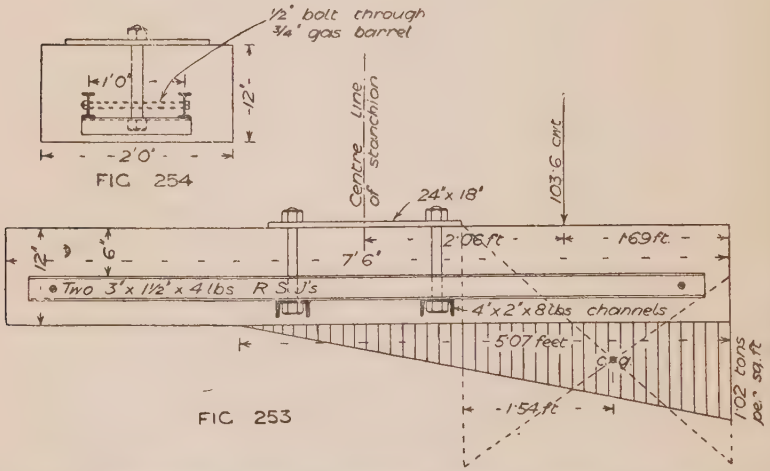


FIG. 253.—Longitudinal section of Reinforced Concrete Foundation for Stanchions.
FIG. 254.—Cross-section of ditto.

overturning force in Fig. 249, giving a resultant cutting the ground at 1·47 ft. from outer edge of base, with a vertical component of 139·5 cwts., or 6·97 tons, spread over a width of 2 ft. 6 ins., or $\frac{6{\cdot}97}{2{\cdot}5} = 2{\cdot}79$ tons per foot wide. As no tension is allowable the formula to adopt in finding the stresses will be $\frac{2}{3} \times \frac{W}{d}$, where W = vertical load in tons and d = distance from resultant to outer edge of base in feet. Then the maximum compression will be $\frac{2}{3} \times \frac{W}{d} = \frac{2}{3} \times \frac{2{\cdot}79}{1{\cdot}46} = 1{\cdot}27$ tons per sq. ft. compression on the soil, which will be quite safe in ordinary

circumstances. The distribution of the stress is shown in Fig. 252, where it varies from a maximum of 1·27 tons per sq. ft. at outer edge to nil at a distance of $3d = 3 \times 1{\cdot}47 = 4{\cdot}41$ ft. from outer edge.

Another method of constructing the base is by reinforced concrete, as shown in Figs. 253 and 254. Assume the size of concrete to be 7 ft. 6 ins. by 2 ft. by 1 ft. deep, weighing $7{\cdot}5 \times 2 \times 1 \times \frac{150}{112} = 20{\cdot}1$ cwts. Add this to the dead load on stanchion, and combine as before, giving a resultant which cuts the base at 1·69 ft. from outer edge, and having a vertical component of 103·6 cwts. = 5·18 tons on a width of 2 ft., or $\frac{5{\cdot}18}{2} = 2{\cdot}59$ tons per foot wide. Then $\frac{2}{3} \times \frac{W}{d} = \frac{2}{3} \times \frac{2{\cdot}59}{1{\cdot}69} = 1{\cdot}02$ tons per sq. ft., which is quite safe, and the distribution will be as shown by the shaded triangle in Fig. 253. To ascertain the amount of reinforcement the base must be taken as a cantilever with an upward pressure tending to turn it about the edge of base-plate. The pressure acting between the edge of base-plate and edge of concrete will be $\frac{1{\cdot}02 + {\cdot}46}{2} \times 2{\cdot}75 \times 20 = 40{\cdot}7$ cwts., which may be taken as a concentrated load acting at the centre of gravity of the pressures, or 1·54 ft. from edge of base-plate, as shown on Fig. 253. The formula to use for the safe load on a reinforced concrete beam supported at both ends under a uniformly distributed load is $W = ({\cdot}37\ p + {\cdot}214)\frac{bd^2}{L}$, where p = percentage of reinforcement, b = breadth of beam in inches, taken as 1 ft. wide in this case, d = depth of beam to centre of reinforcement in inches, L = span in feet; but as this is for a cantilever with concentrated load, only one-eighth of the load could be carried, or $W = ({\cdot}37\ p + {\cdot}214)\ \frac{bd^2}{8\ L}$, whence $8\ WL = ({\cdot}37\ p + {\cdot}214)\ bd^2$, or $8 \times 40{\cdot}7 \times 1{\cdot}54 = ({\cdot}37\ p + {\cdot}214)\ 12 \times 7{\cdot}5^2$, or $501{\cdot}42 = 249{\cdot}75\ p + 144{\cdot}45$, and $p = \frac{501{\cdot}42 - 144{\cdot}45}{249{\cdot}75}$ = 1·43 per cent. The percentage is taken on the area of the concrete above the centre of the reinforcement and not on the

whole area of the beam. This will require an area of reinforcement of $\frac{1{\cdot}43 \times 7{\cdot}5 \times 12}{100} = 1{\cdot}287$ sq. in. per foot wide, say 3-in. by $1\frac{1}{2}$-in. by 4-lb. rolled-steel joists 12-in. centres. The area provided does not appear to be quite sufficient, but the formula adopted allows merely for the sectional area of reinforcement, whilst the rolled joist section adopted adds independent stiffness. The holding-down bolts may be bolted to channel sections underneath the rolled joist reinforcement.

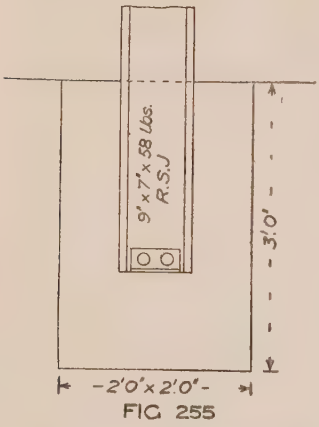


FIG 255

FIG. 255.—Usual construction of Foundation for Shed Stanchions.

(c) The calculation of the resistance offered to movement by this method is beset with difficulties. From a rough attempt at analysis it appears to be somewhere about $6d^3$ in lb.-ft. per foot where d is depth in feet. So that given the overturning moment in lb.-ft. as M, and the width in feet opposed to movement as b, the requisite depth from ground-level will be $M = 6\ bd^3$, or $d = \sqrt[3]{\frac{M}{6\ b}}$. Applying this formula to the present case, say $b = 3$ ft., thickness half the breadth, and $M = 10{\cdot}7 \times 2{,}240 = 23{,}968$ lb.-ft., then $d = \sqrt[3]{\frac{23{,}968}{6 \times 3}} = 11$ ft. deep, which is impossible. Try an increased width, say $b = 5$ ft., then thickness $= 2$ ft. 6 ins., $d = \sqrt[3]{\frac{23{,}968}{6 \times 5}} = 9{\cdot}28$ ft. deep, requiring $9{\cdot}28 \times 5 \times 2{\cdot}5 = 116$ cub. ft. of concrete, as against $6 \times 2{\cdot}5 \times 3 = 45$ cub. ft. when placed as in Fig. 252. Commonly this method takes the form of Fig. 255, which is useless so far as stability against wind-pressure is concerned.

CHAPTER XXIII

Details of Roof Trusses for Steel-Framed Shed—General Elevation of Framing

Continuing the calculations for the steel-framed shed, the stresses and scantlings of the roof truss having been worked out in Chapter XXI., the joints may now be designed. The lines of the frame diagram Fig. 242 represent the centre of gravity lines for the various members, except at the foot of principal rafter, where it is more convenient to put in a strong joint plate and permit secondary stresses, which, however, will only be of trifling amount. The other joint plates will also be better if made $\frac{3}{8}$ in., thick, as the stress is not quite direct in cross-section, owing to the joint plates being put on one side of the web of tee, whereas, had the stress been direct, $\frac{5}{16}$ in. would have been sufficient. It will be an aid to the allotment of the correct number of rivets to each joint to make a short list of the resistance provided by any given number, thus :—

$\frac{3}{4}$-in. rivets single shear,

$1 = 44 \times 1 = 44$ cwts
$2 = 44 \times 2 = 88$ „
$3 = 44 \times 3 = 132$ „
$4 = 44 \times 4 = 176$ „
$5 = 44 \times 5 = 220$ „

And $\frac{3}{4}$-in. rivets in double shear,

$1 = 44 \times 1\frac{1}{2} \times 1 = 66$ cwts.
$2 = 44 \times 1\frac{1}{2} \times 2 = 132$ „
$3 = 44 \times 1\frac{1}{2} \times 3 = 198$ „

Then, having drawn out the various members meeting at each joint, the rivets are to be marked on at a suitable pitch, say from $2\frac{1}{2}$ ins. to $3\frac{1}{2}$ ins., the outlines of the joint plates will be added as shown in Figs. 256 to 258.

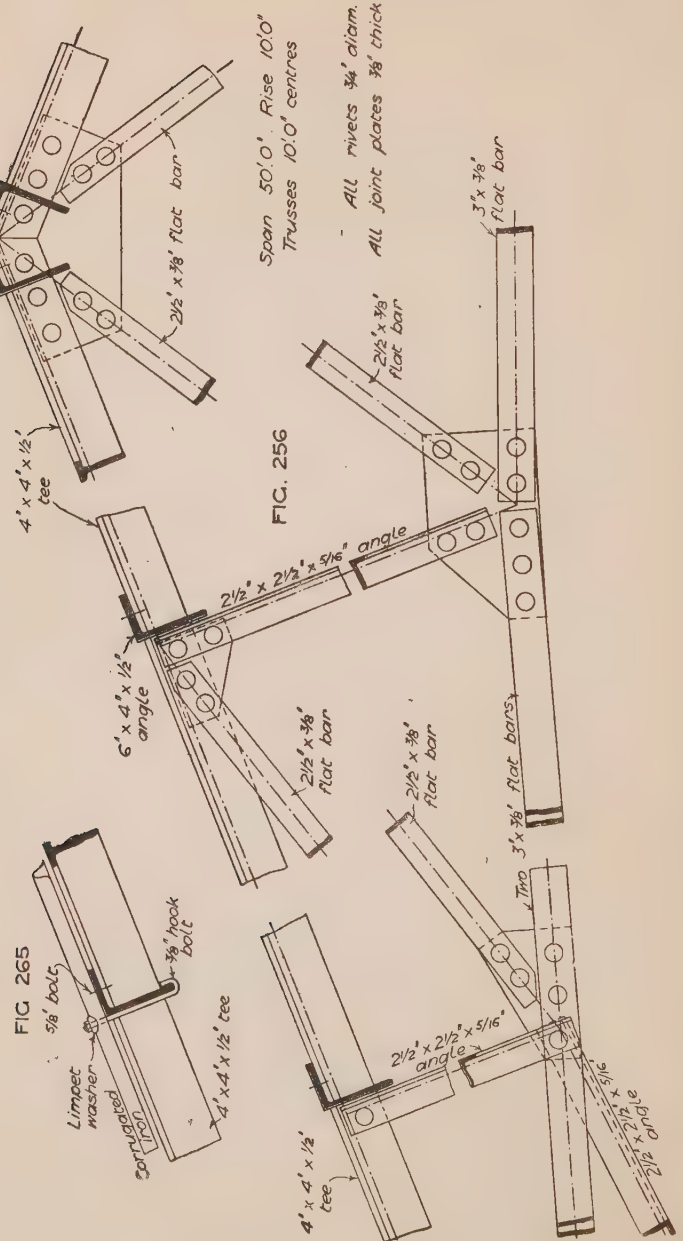


FIG. 256.—Details of various Joints of the Roof Truss.

The horizontal angle bars at the sides of the shed to support the corrugated iron between the stanchions must now be calculated. At 25 lbs. per sq. ft. they will have to resist a wind pressure of, say, $\frac{5\frac{2}{3} \times 10 \times 25}{2{,}240} = 0{\cdot}63$ ton, distributed over a span of 10 ft., producing a maximum bending moment of $\frac{0{\cdot}63 \times 10 \times 12}{8} = 9{\cdot}45$ ton-ins. Try a 4-in. by 3-in. by $\frac{3}{8}$-in. angle, then $x = 1{\cdot}2$-in. and $y = 2{\cdot}8$-ins., the moment of inertia $I = 4{\cdot}42$, therefore $Z_x = \frac{4{\cdot}42}{1{\cdot}2} = 3{\cdot}68$, and $Z_y = \frac{4{\cdot}42}{2{\cdot}8} =$

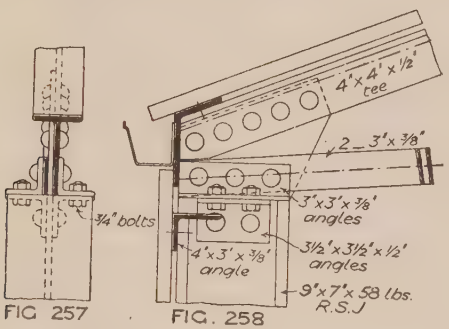


FIG. 257.—End elevation of Joint at Support. FIG. 258.—Front elevation of Joint at Support.

1·58, and the stresses will be $\frac{M}{Z_x} = \frac{9{\cdot}45}{3{\cdot}68} = 2{\cdot}57$ tons per sq. in. compression in the short side, and $\frac{M}{Z_y} = \frac{9{\cdot}45}{1{\cdot}58} = 6$ tons per sq. in. tension in the long side. Besides the wind pressure, these angles will have to resist the weight of the sheeting and their own weight. The weight of the sheeting will be $10 \times 5\frac{2}{3} \times 2$ lbs. per sq. ft. = 113·3 lbs. on each, and the angle itself will weigh $10 \times 8{\cdot}45 = 84{\cdot}5$ lbs., together $113{\cdot}3 + 84{\cdot}5 = 197{\cdot}8$ lbs., or, say, $\frac{200}{2{,}240} = {\cdot}09$ of a ton, producing a bending moment of $\frac{{\cdot}09 \times 10 \times 12}{8} = 1{\cdot}35$ ton-ins. The stresses induced by this load will be found by taking the 4-in. by 3-in. angle in

the other direction, where $x = 0{\cdot}78$ in., $y = 2{\cdot}22$ ins., and the moment of inertia $= 1{\cdot}92$. Then $Z_x = \frac{1{\cdot}92}{0{\cdot}78} = 2{\cdot}46$, and $Z_y = \frac{1{\cdot}92}{2{\cdot}22} = 0{\cdot}87$, and the stresses produced will be $\frac{M}{Z_x} = \frac{1{\cdot}35}{2{\cdot}46}$ $= 0{\cdot}55$ ton per sq. in. compression in the long side, and $\frac{1{\cdot}35}{0{\cdot}87} =$ $1{\cdot}67$ tons per sq. in. tension in the short side. These stresses will act in opposition to those produced by the wind pressure in the outer edges of the flanges, but in conjunction with them at the root. The ultimate stresses will therefore be $2{\cdot}57 + 0{\cdot}55 =$

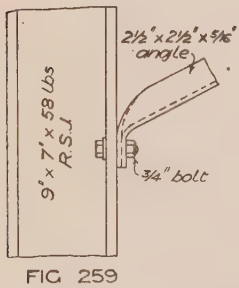
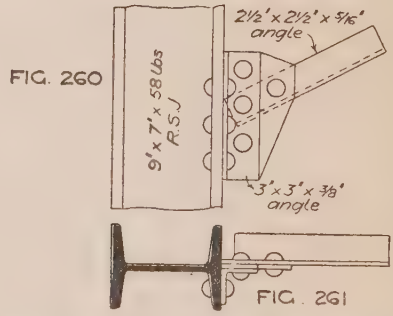


FIG. 259.—Joint between Stanchion and Brace from Roof Truss.



FIGS. 260 and 261.—Alternative form of Joint between Stanchion and Brace from Roof Truss.

$3{\cdot}12$ tons per sq. in. compression at the root, and $6 - 0{\cdot}55 =$ $5{\cdot}45$ tons per sq. in. tension in the outer edge of long side, and $2{\cdot}57 - 1{\cdot}67 = 0{\cdot}9$ ton per sq. in. compression in the outer edge of short side, which will be satisfactory. The angle brace between roof truss and stanchion was calculated to be $2\frac{1}{2}$ ins. by $2\frac{1}{2}$ ins. by $\frac{5}{16}$ ins. They are very often made much larger, but that is because they are dimensioned without calculation. A simple method of fixing these braces is shown in Fig. 259, and an alternative method for larger braces in Figs. 260 and 261.

The end elevation of shed is shown in Fig. 262 and the side elevation in Fig. 263. The longitudinal diagonal braces, being longer, may be 3 ins. by 3 ins. by $\frac{3}{8}$ in. with the ends closed down and bolted to stanchions, as in Fig. 259, but to the webs

instead of to the flanges. The end of shed will be supported by stanchions and angle bars, but without diagonal bracing as this is provided in both directions on the long sides. The wind may be taken at 21 lbs. per foot super over the end, straining the

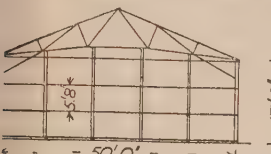


FIG. 262.

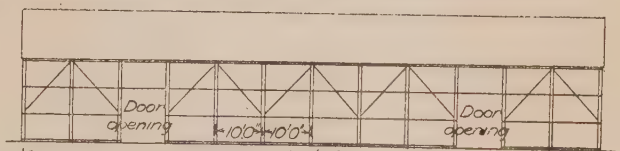


FIG. 263.

FIG. 262.—End elevation of Shed. FIG. 263.—Side elevation of Shed.

stanchions as cantilevers under a distributed load. The calculation for one near the centre will be as follows: The total wind pressure $= 19{\cdot}5 \times 10 \times 21 = 4{,}095$ lbs.; whence the maximum bending moment will be $\frac{WL}{2} = \frac{4{,}095 \times 19{\cdot}5}{2} = 39{,}926$ lb.-ft. $= \frac{39{,}926 \times 12}{2{,}240} = 213{\cdot}9$, say 214 ton-ins. Try an 8-in. by 6-in. by 35 lbs. rolled steel joist with a section modulus of 27·62, then $\frac{M}{Z} = \frac{214}{27{\cdot}62} = 7{\cdot}7$ tons per sq. in., which is a little

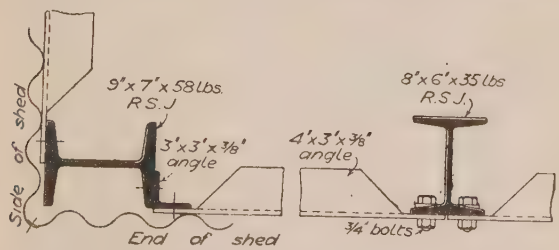


FIG. 264.

FIG. 264.—Detail showing Corner Stanchion and one of the End Stanchions.

high, but this section may be adopted. Fig. 264 shows the corner stanchion at one end of a long side and one of the end stanchions adjacent. The foundations for the latter will be as already calculated, or something rather less may be provided, in the ratio of the section modulus of the two stanchions.

Galvanised corrugated sheet-iron is made in sheets 6 ft. or 8 ft. long, 2 ft. 6 ins. wide after corrugation, with 5-in. corruga-

tions, having a depth of $\frac{1}{4}$ the width. It should be laid with 6-in. laps when on the slope and 3-in. laps when vertical. Laps to break joint by 18 ins. Hook bolts to purlins, as Fig. 265, (p. 178) about 2 ft. apart, with limpet washers. The thicknesses vary from No. 16 to No. 26 gauge (B.G.), the former for first-class work, and the latter for shipment abroad. A town atmosphere rapidly destroys corrugated iron, but its decay may be retarded by painting. A smaller corrugation is made, viz., $2\frac{1}{2}$ ins. by $\frac{5}{8}$ in., but it is not much used except for very light work, such as covering fowl runs, etc.

CHAPTER XXIV

Designing Stanchions for more than one Storey—Comparison of Methods—Impossibility of Avoiding Laborious Calculations

An example will now be taken of the calculations for a stanchion in a three-storeyed building. The conditions of loading are shown in Fig. 265. There are two alternatives: either a stanchion may be selected sufficiently large to carry the load at the bottom storey with the section uniform throughout, or a section sufficient for the top storey may be found, which may be strengthened by adding plates to meet the additional load at each storey below. It is feasible to suppose that a fairly deep rolled joist will be desirable for the stanchion to give leverage in the cross section against the bending moment produced by the eccentric loading due to floors being on one side of stanchion only.

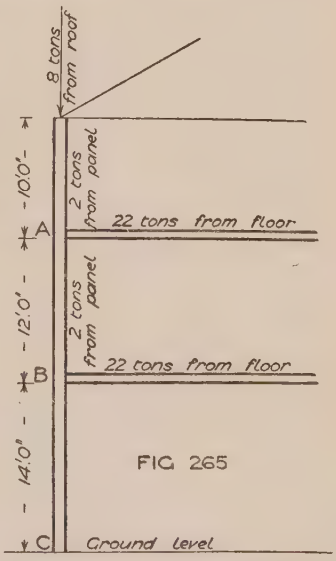


Fig. 265.—Sketch showing Loads upon Stanchion.

The diagram, Fig. 265, shows a 14-in. by 6-in. by 57-lb. rolled-steel joist as a stanchion, with axial loads from the roof and front wall panels and non-axial loads from two floors. By consulting standard tables it is found that this section has an area of 16·76 sq. ins., a section modulus of 76·12 inch units, and least radius of gyration of 1·3 ins. From the top of the stanchion down to point A there will be a uniform stress of $\frac{W}{A} = \frac{8+2}{16{\cdot}76} =$

0·6 ton per sq. in. compression over the whole section. The non-axial load of 22 tons produces a bending moment of $22 \times \frac{14}{2}$ = 154 ton-ins., and the extra stress produced between A and B $= \frac{W}{A} \pm \frac{M}{Z} = \frac{22}{16{\cdot}76} \pm \frac{154}{76{\cdot}12} = 1{\cdot}43 \pm 2{\cdot}02 = +3{\cdot}45$ and $-$ 0·59 tons per sq. in., to which must be added the compression

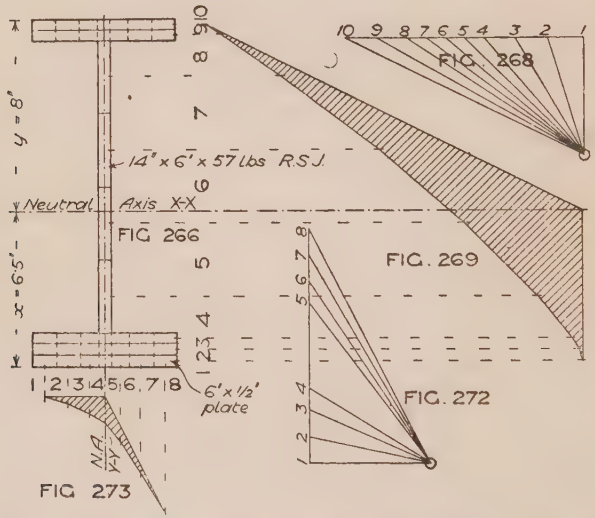


Fig. 266.—Section of Rolled-steel Joist with Plate attached to one Flange. Fig. 268.—Vector diagram for Fig. 266. Fig. 269.—Maximum inertia area for ditto. Fig. 272.—Vector diagram for Fig. 266. Fig. 273.—Minimum inertia area for ditto.

of 0·6 ton, making a total stress of 4·05 tons per sq. in. compression at inner edge, and 0·01 ton per sq. in. compression at outer edge. From B to C the extra stress will be $\frac{W}{A} \pm \frac{M}{Z} = \frac{22}{16{\cdot}76} \pm \frac{154}{76{\cdot}12} = 1{\cdot}31 \pm 2{\cdot}02 = +3{\cdot}33$ tons per sq. in., and $-$ 0·71 ton per sq. in., and the final stresses at ground level will be 7·38 tons per sq. in. compression at inner edge and 0·7 ton per sq. in. tension at outer edge.

By the L. C. C. rule the maximum allowable stress for

the ratio of $\frac{l}{r} = \frac{14 \times 12}{1{\cdot}3} = 129{\cdot}22$ is 3·27 tons per sq. in. so that this section would not be passed as safe. Add a 6-in. by ½-in. plate to the inner edge, as Fig. 266, from the second floor level to ground, and another from first floor to ground, as Fig. 267, and as the section modulus cannot be readily calculated, the section must be drawn out as in Fig. 266, and the maximum

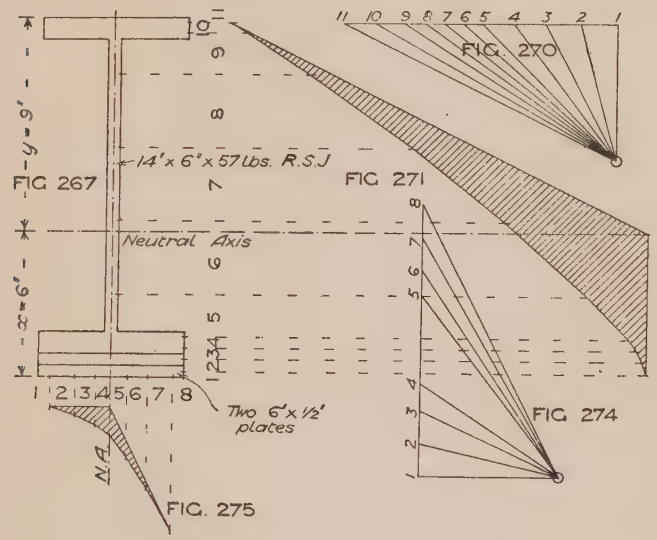


FIG. 267.—Section of Rolled-steel Joist with two 6-in. by ½-in. Plates attached to one Flange. FIG. 270.—Vector diagram for Fig. 267. FIG. 271.—Maximum inertia area for ditto. FIG. 274.—Vector diagram for Fig. 267. FIG. 275.—Minimum inertia area for ditto.

inertia area found graphically as follows. Divide the section into any convenient number of parts, and draw horizontal lines through the centre of gravity of each part. Calculate the separate areas, and set them down as a load line 1 to 10 in Fig. 268. Set off pole 0 at right angles to 1-10 and equal to half the distance from 1 to 10. Draw vectors and parallel with these across their respective spaces construct the inertia area, Fig. 269, the intersection of the lines parallel with 0—1 and 0—10 giving a point on the neutral axis. This area is then measured by planimeter, and allow-

ance made for scale—that is, if drawn $\frac{1}{4}$ full-size, the area found must be multiplied by $(4)^2 = 16$, to give the full-size area. Then $I = Aa$ where A = area of section and a = inertia area, and in the present case $I =$

$Aa = (16{\cdot}76 + 6 \times \frac{1}{2}) \times 34{\cdot}4 = 679{\cdot}8$, $Zx = \frac{679{\cdot}8}{6{\cdot}5} = 104{\cdot}6$, $Zy = \frac{679{\cdot}8}{8} = 84{\cdot}97$, and $M = 22 \times 6{\cdot}5 = 143$ ton-ins. for the part from second to first floor; and for the part from first floor to ground it is found from Figs. 270 and 271 that $I = (16{\cdot}76 + 2 \times 6 \times \frac{1}{2}) \times 35{\cdot}2 = 801$, $Zx = \frac{801}{6} = 133$, $Zy = \frac{801}{9} = 89$, and $M = 22 \times 6 = 132$ ton-ins. Then, as before, the stress from roof to point A = 0·6 ton per sq. in. compression; the extra stress between A and B $= \frac{W}{A} + \frac{M}{Z_x} = \frac{22 + 2}{19{\cdot}76} + \frac{143}{104{\cdot}6} = 1{\cdot}21 + 1{\cdot}37 = 2{\cdot}58$ tons per sq. in. compression, and $\frac{W}{A} - \frac{M}{Z_y} = \frac{22 + 2}{19{\cdot}76} - \frac{143}{84{\cdot}97} = 1{\cdot}21 - 1{\cdot}68 = 0{\cdot}47$ ton per sq. in. tension; and the extra stress between B and C $= \frac{W}{A} + \frac{M}{Z_x} = \frac{22}{22{\cdot}76} + \frac{132}{133} = 0{\cdot}97 + 1{\cdot}0 = 1{\cdot}97$ tons per sq. in. compression, and $\frac{W}{A} - \frac{M}{Z_y} = \frac{22}{22{\cdot}76} - \frac{132}{89} = 0{\cdot}97 - 1{\cdot}48 = 0{\cdot}51$ ton per sq. in. tension, making the total stress at B 3·18 tons per sq. in. compression at inner edge and 0·13 ton per sq. in. compression at outer edge, and the total stress at C 5·15 tons per sq. in. compression and 0·38 ton per sq. in. tension. To find the least radius of gyration to ascertain the L. C. C. allowance for stress, obtain the inertia area for bending in the other direction by means of Figs. 272 and 273, then $r = \sqrt{a} = \sqrt{2{\cdot}56} = 1{\cdot}6$. The ratio $\frac{l}{r}$ at B $= \frac{12 \times 12}{1{\cdot}6} = 90$, or 4·25 tons per sq. in., and at C by means of Figs. 274 and 275 $r = \sqrt{3{\cdot}04} = 1{\cdot}74$, $\frac{14 \times 12}{1{\cdot}74} = 96{\cdot}55$, or 4·09 tons per sq. in. It will therefore be seen that the addition of the one $\frac{1}{2}$-in. plate will be sufficient between A and

B, but the second $\frac{1}{2}$-in. plate between B and C is not sufficient, and something more substantial is necessary. Try the addition of a 9-in. by $3\frac{1}{2}$-in. by 25·4-lb. channel instead of the second $\frac{1}{2}$-in. plate, as shown in the section Fig. 276. The inertia area

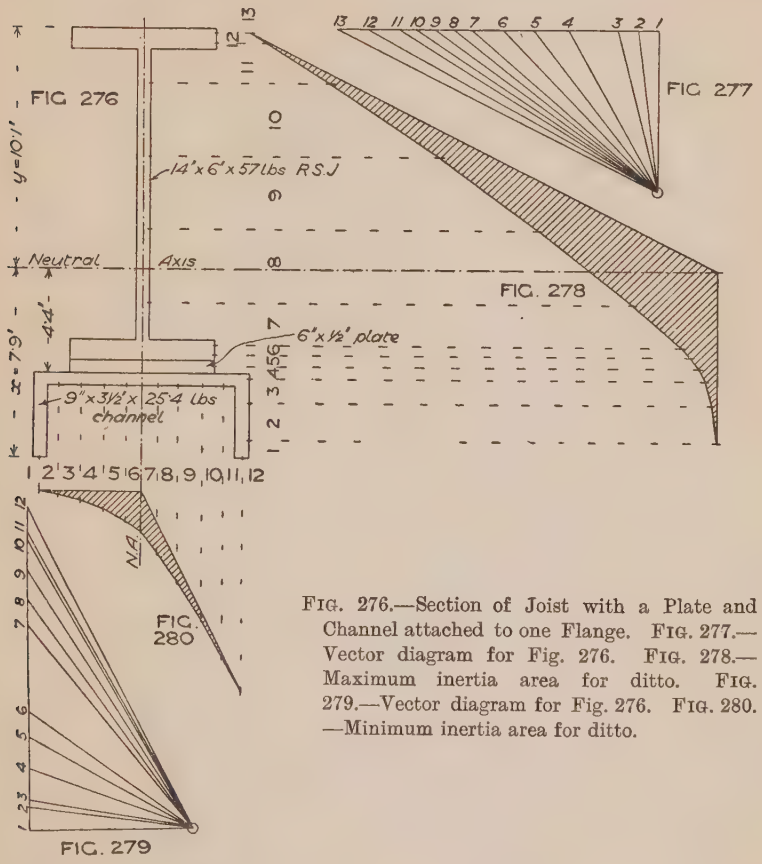


FIG. 276.—Section of Joist with a Plate and Channel attached to one Flange. FIG. 277.—Vector diagram for Fig. 276. FIG. 278.—Maximum inertia area for ditto. FIG. 279.—Vector diagram for Fig. 276. FIG. 280.—Minimum inertia area for ditto.

may be found by means of Figs. 277 and 278, and $I = Aa = 27{\cdot}23 \times 38{\cdot}24 = 1041{\cdot}3$, $Zx = \frac{1041{\cdot}3}{7{\cdot}9} = 132$, $Zy = \frac{1041{\cdot}3}{10{\cdot}1} = 103$. $M = 22 \times 4{\cdot}4 = 96{\cdot}8$ ton-ins. The extra stress between B and C $= \frac{W}{A} + \frac{M}{Z_x} = \frac{22}{27{\cdot}23} + \frac{96{\cdot}8}{132} = 0{\cdot}8 + 0{\cdot}73 = 1{\cdot}53$ tons

per sq. in. compression and $\frac{W}{A} - \frac{M}{Z_y} = \frac{22}{27{\cdot}23} - \frac{96{\cdot}8}{108} = 0{\cdot}8 - 0{\cdot}94 = 0{\cdot}14$ ton per sq. in. tension, and the total stress will be 4·71 tons per sq. in. compression at inner edge, and 0·01 ton per sq. in. tension at outer edge. The least radius of gyration found from Figs. 279 and 280 $= \sqrt{6{\cdot}24} = 2{\cdot}5$, and the ratio $\frac{l}{r} = \frac{14 \times 12}{2{\cdot}5} = 67{\cdot}2$, or, say, 4·75 tons per sq. in., so that the section proposed will be within safe limits. The advantages of extra area and side-flanges to the stanchion produced by the addition of a channel section are, to a certain extent, reduced by the increased distances of x and y to the edges of the section, and the section moduli do not increase in proportion to the inertia area.

This does not seem a satisfactory conclusion, and it will be desirable to try a section with broader flanges and less depth, say the 9 ins. by 7 ins. by 58 lbs. It will be observed that it is 5 ins. less in depth than the section already tried, although only 1 lb. per ft. less in weight and the reduction of depth will give less overhanging to the side loads reducing the maximum stress, while the extra width of flange will increase the least radius of gyration, and permit of higher stresses being adopted. The 9-in. by 7-in. by 58-lb. rolled steel joist has an area of 17·06 sq. ins., a section modulus of = 51·0, and a least radius of gyration of 1·64. From top of stanchion to point A the stress will be $= \frac{W}{A} = \frac{8 + 2}{17{\cdot}06} = 0{\cdot}58$ ton per sq. in. compression. Between A and B the bending moment $= 22 \times \frac{9}{2} = 99$ ton-ins., and the extra stress between A and B $= \frac{W}{A} \pm \frac{M}{Z} = \frac{22 + 2}{17{\cdot}06} \pm \frac{99}{51} = 1{\cdot}41 \pm 1{\cdot}94 = 3{\cdot}35$ tons per sq. in. compression and 0·53 ton per sq. in. tension, and the total stress at B $= 3{\cdot}35 + 0{\cdot}58 = 3{\cdot}93$ tons per sq. in. compression at inner edge and $0{\cdot}58 - 0{\cdot}53 = 0{\cdot}05$ ton per sq. in. compression at outer edge.

The extra stress between B and C $= \frac{W}{A} \pm \frac{M}{Z} = \frac{22}{17{\cdot}06} \pm \frac{99}{51}$

$= 1{\cdot}26 \pm 1{\cdot}94 = 3{\cdot}2$ tons per sq. in. compression and 0·65 ton per sq. in. tension, and the total stress at C $= 3{\cdot}2 + 3{\cdot}93 = 7{\cdot}13$ tons per sq. in. compression at inner edge and $-0{\cdot}65 + 0{\cdot}05 = 0{\cdot}6$ ton per sq. in. tension at outer edge.

By the L. C. C. rule the allowable stress for 12 ft. high $= 6{\cdot}5 - \frac{1}{40} \times \frac{12 \times 12}{1{\cdot}54} = 4{\cdot}8$ tons per sq. in. and for 14 ft. high $= 6{\cdot}5 - \frac{1}{40} \times \frac{14 \times 12}{1{\cdot}54} = 3{\cdot}9$ tons per sq. in., showing that the joist alone will not be sufficient between B and C.

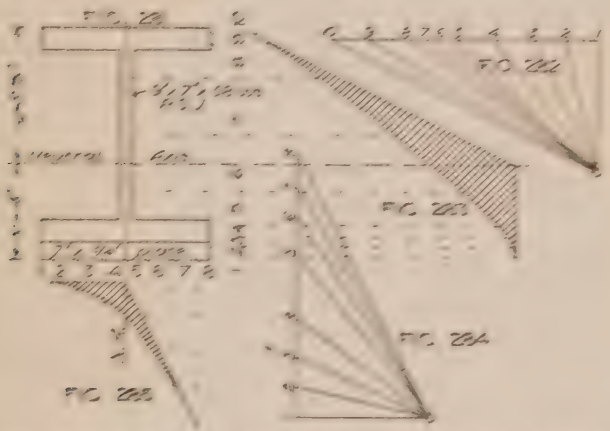


Fig. 281.—Section of 8-in. by 7-in. by 19-lb. Rolled-steel Joist with Plate attached to one Flange. Fig. 282.—Stress diagram for Fig. 281. Fig. 283.—Maximum inertia axes for ditto. Fig. 284.—Stress diagram for Fig. 281. Fig. 285.—Minimum inertia axes for ditto.

With a 7-in. by $\frac{3}{4}$-in. plate added between B and C as in Fig. 281, making area 22·31 sq. ins., I from Figs. 282 and 283 $= = Aa = 22{\cdot}31 \times 14{\cdot}93 = 331{\cdot}37$, say 332, $M = 22 \times 4{\cdot}9 = 88$ ton-ins. $Z_x = \frac{332}{4} = 83$, and $Z_y = \frac{332}{5{\cdot}75} = 57{\cdot}7$.

Then the extra stress between B and C $= \frac{W}{A} + \frac{M}{Z_x} = \frac{22}{22{\cdot}31} + \frac{88}{83} = 1{\cdot}0 + 1{\cdot}06 = 2{\cdot}06$ tons per sq. in. compression and $\frac{W}{A} - \frac{M}{Z_y} = \frac{22}{22{\cdot}31} - \frac{88}{57{\cdot}7} = 1{\cdot}0 - 1{\cdot}52 = 0{\cdot}52$ ton per sq. in.

tension. Total stress at C = 3·95 + 2·06 = 6·01 tons per sq. in. compression and ·07 − 0·52 = 0·45 tons per sq. in. tension. From Figs. 284 and 285 the least radius of gyration $r = \sqrt{a} = \sqrt{3{\cdot}68} = 1{\cdot}92$, and the London County Council allowable stress $= 6{\cdot}5 - \frac{1}{40} \times \frac{14 \times 12}{1{\cdot}92} = 4{\cdot}31$ tons per sq. in. against the 6·01 tons which would occur.

Instead of endeavouring to bring this single joist up to the required strength by extra plates, it is evident that a fundamental departure is necessary, and a compound section with two rolled

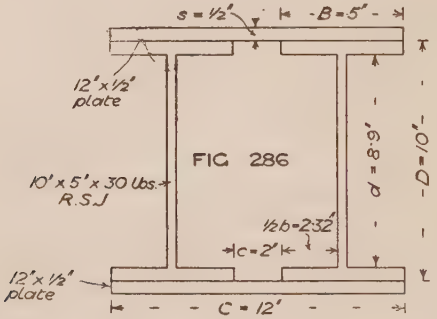


Fig. 286.—Section of Compound Stanchion composed of two Rolled-steel Joists with Plate on either side.

joists and plates must be tried. Try two 10-in. by 5-in. by 30-lb. rolled steel joists connected by two 12-in. by ½-in. plates as in Fig. 286, and for instructive practice calculate by formula the moment of inertia in the two directions.

The moment of inertia about $XX =$

$$\frac{2\,(BD^3 - bd^3) + C\,(D + 2s)^3 - CD^3}{12} =$$

$$\frac{2\,(5 \times 10^3 - 4{\cdot}64 \times 8{\cdot}9^3) + 12\,(10 + 1)^3 - 12 \times 10^3}{12} =$$

620, and $Z = \frac{620}{5{\cdot}5} = 112{\cdot}7$. The moment of inertia about YY

$$= \frac{(D - d)\,(C^3 - c^3) + d\,(C - b)^3 + 2s\,C^3 - d\,(b + c)^3}{12} =$$

$$\frac{1{\cdot}1\,(12^3 - 2^3) + 8{\cdot}9\,(12 - 4{\cdot}64)^3 + 2 \times 0{\cdot}5 \times 12^3 - 8{\cdot}9\,(4{\cdot}6 + 2)^3}{12}$$

$$= 301{\cdot}5, \text{ and } r = \sqrt{\frac{I}{A}} = \sqrt{\frac{301{\cdot}5}{29{\cdot}6}} = 3{\cdot}2 \text{ ins.}$$

From top of stanchion to point A, the stress will be $\frac{W}{A} = \frac{8+2}{29{\cdot}6}$ $= 0{\cdot}34$ ton per sq. in. Between A and B the bending moment $= 22 \times \frac{11}{2} = 121$ ton-ins., and the extra stress $= \frac{W}{A} \pm \frac{M}{Z} =$ $\frac{22+2}{29{\cdot}6} \pm \frac{121}{112{\cdot}7} = 0{\cdot}81 \pm 1{\cdot}07 = 1{\cdot}88$ tons per sq. in. compression and $0{\cdot}26$ ton per sq. in. tension, making the total stress at $B = 1{\cdot}88 + 0{\cdot}34 = 2{\cdot}22$ tons per sq. in. compression and $- 0{\cdot}26 + 0{\cdot}34 = 0{\cdot}08$ tons per sq. in. compression. The London County Council allowable stress $= 6{\cdot}5 - \frac{1}{40} \times \frac{12 \times 12}{3{\cdot}2} =$ $5{\cdot}38$ tons per sq. in. Between B and C the extra stress $= \frac{W}{A} \pm$ $\frac{M}{Z} = \frac{22}{29{\cdot}6} \pm \frac{121}{112{\cdot}7} = 0{\cdot}74 \pm 1{\cdot}07 = 1{\cdot}81$ tons per sq. in. compression, $0{\cdot}33$ ton per sq. in. tension, and the total stress at C $= 1{\cdot}81 + 2{\cdot}22 = 4{\cdot}03$ tons per sq. in. compression and $- 0{\cdot}33$ $+ 0{\cdot}08 = 0{\cdot}25$ ton per sq. in. tension. London County Council allowable stress $= 6{\cdot}5 - \frac{1}{40} \times \frac{14 \times 12}{3{\cdot}2} = 5{\cdot}2$ tons per sq. in. It is possible that $\frac{3}{8}$-in. plates might be substituted for the $\frac{1}{2}$-in. plates, but the allowable stress would then be smaller and the actual stress greater, while if $\frac{1}{2}$-in. plates are retained, the margin of strength will not be excessive.

CHAPTER XXV

Use of Substituted Members in Finding Reciprocal Stress Diagrams—Examples for Various Types of Roof Truss

In determining the stresses on braced structures by means of reciprocal diagrams a difficulty often arises by reason of the existence of superfluous members. Although redundant so

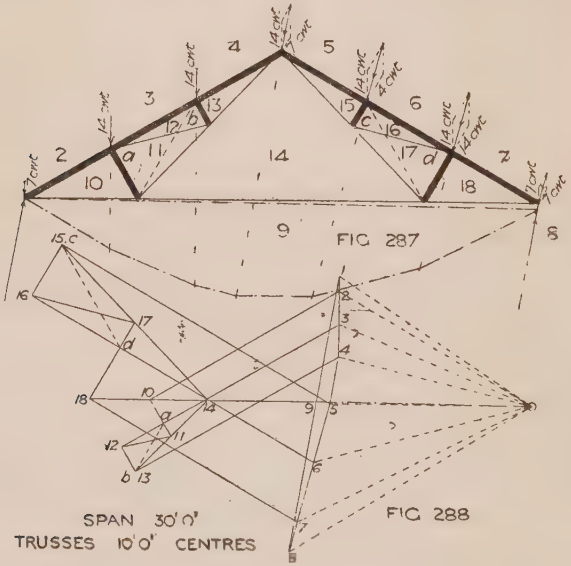


Fig. 287.—Frame diagram of Roof Truss with substituted Members.
Fig. 288.—Stress diagram for ditto.

far as the necessary transmission of stress is concerned, they are generally desirable from constructional or practical considerations, and hence it is necessary to accept the conditions and find the stresses that occur in all the members under the given loading. To do this involves the expedient of assuming

temporarily certain substituted members to simplify the truss. The method was first shown by Mr. James R. Willett, in a paper on "Some Applications of Graphical Statics," read before the Chicago Chapter of the American Institute of Architects, on March 22, 1888. He kindly sent the author a copy of the paper

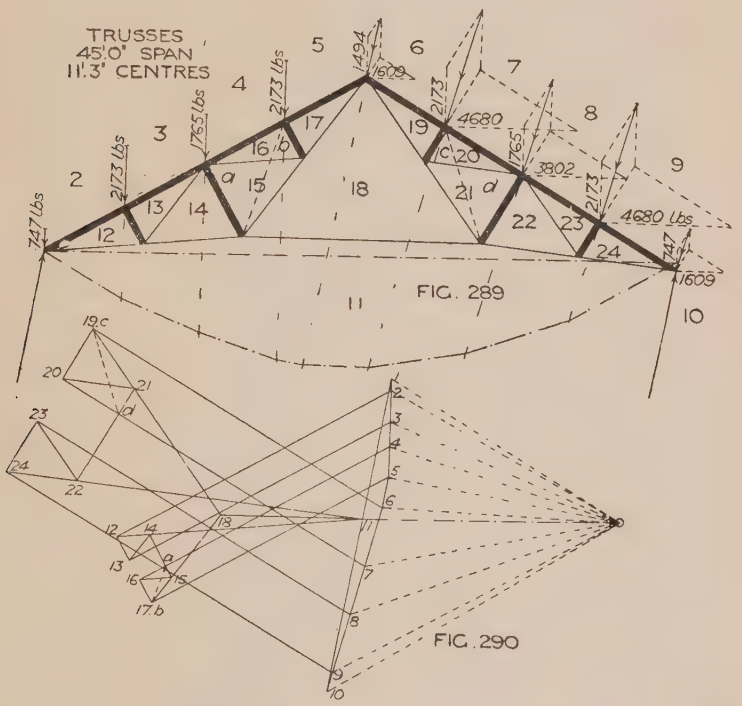


FIG. 289.—Fink Truss with substituted Members.
FIG. 290.—Stress diagram for ditto.

through Mr. R. H. Bow, of Edinburgh, who is well known as the originator of Bow's Notation. In the ordinary way the points of a stress diagram are obtained by working from two known points to find the unknown third point, but this fails when the truss contains internal triangulation. For example, in the truss Fig. 287 a substituted member *a-b*, as shown by the dotted line, must be inserted in the place of 11-12 and 12-13, in order to find point 14; thus, after point 10 in the stress diagram Fig. 288, the

order of working will be 10-*a*, 3-*a*, *a*-*b*, 4-*b*; *b*-14, 9-14; then, ignoring points *a* and *b*, the order of working will be 10-11, 14-11; 11-12, 3-12; 12-13, 4-13. A similar substitution is made on the other side of the truss.

The theory is that the principal stresses will be the same whatever arrangement be adopted for the internal trussing, provided the points of application of the loads and reactions are unaltered.

The ordinary Fink truss, Fig. 289, may be worked in a similar

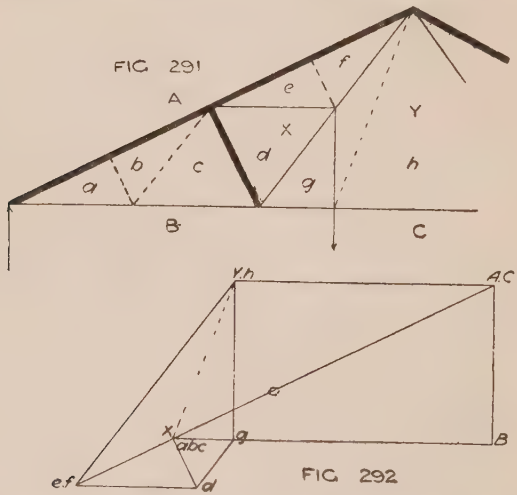


FIG. 291.—Part frame diagram of Truss with Loads hanging from inner Joints.
FIG. 292.—Stress diagram for ditto.

manner. The stress diagram is shown in Fig. 290, and no difficulty will be found after the previous explanation. Fig. 291 shows the case of a Fink truss that appeared in the *Engineering Record*, where it was required to determine the additional stresses produced in the members by certain hanging loads concentrated at the inner joints of the upper short struts. Inserting a substituted member as shown by the dotted line, from the apex to the tie-bar, the stress diagram Fig. 292 is commenced by drawing the load line *CB* and reaction *BA* coinciding with it. Then, assuming all the inner truss members to the left of the dotted line to be absent, *A*-*X*, *B*-*X* gives

point *X*, which will also be *a b* and *c*; *X-Y*, *C-Y* gives point *Y*, which is *h*. Then *h-g* B-*g* gives point *g*; *g-d*, *abc-d*, gives point *d*; *A-ef*, *d-ef*, gives point *ef*; and *ef-h* completes the half-diagram. There will be no stress from the load in those members which are dotted, viz., *a-b*, *b-c*, and *e-f*. Fig. 293 shows a truss

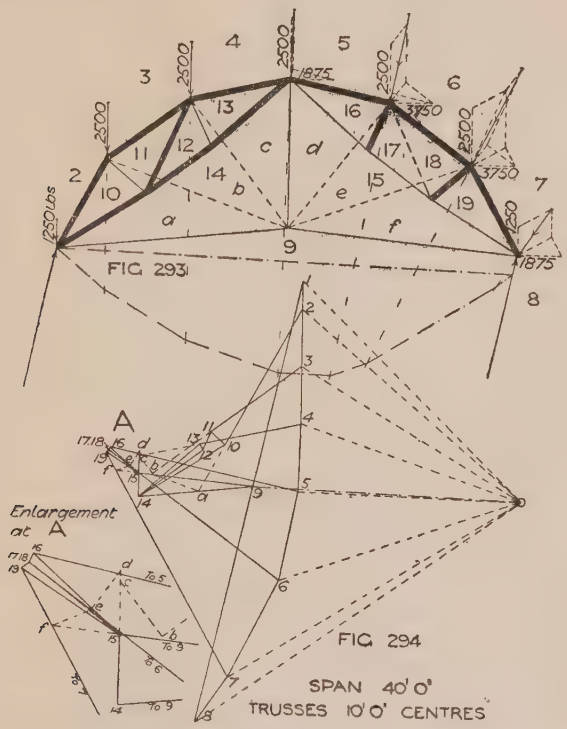
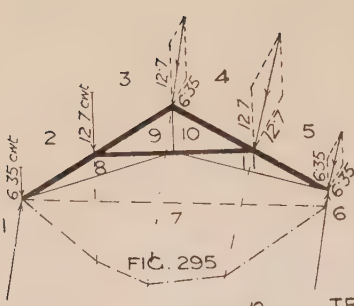
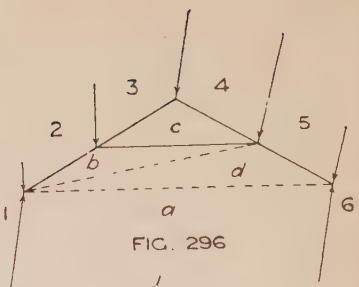


FIG. 293.—Frame diagram of Circular Truss with substituted Members.
FIG. 294.—Stress diagram for ditto.

that presents rather more difficulty. Find the reactions by means of a funicular polygon in the ordinary way, then draw the load line in Fig. 294, and from point 9 draw 9-14 and 9-15 indefinitely. Complete the diagram for the dotted members and produce *c-d* to meet the lines parallel to 9-14 and 9-15, giving the points 14 and 15. Then ignoring points *abcdef*, the full stress diagram may be completed.





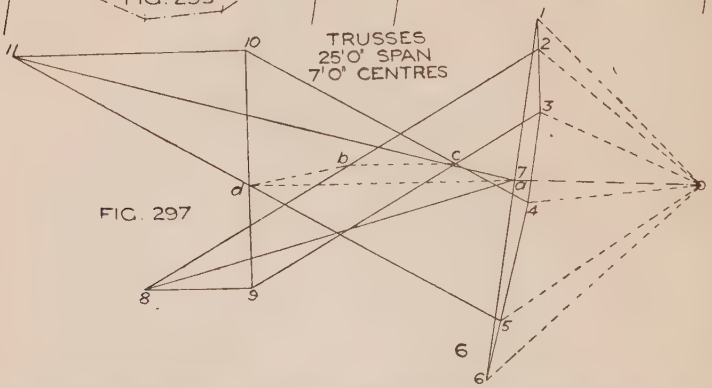


FIG. 295.—Frame diagram of Truss with Reactions found by Funicular Polygon. FIG. 296.—Ditto, with Reactions found by substituted Members. FIG. 297.—Stress diagram for ditto.

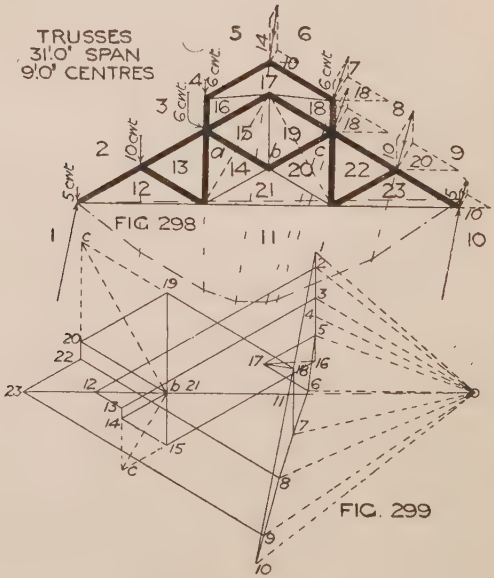


FIG. 298.—Frame diagram of Truss with Lantern Light.
FIG. 299.—Stress diagram for ditto.

A common form of roof truss is shown in Fig. 295, the reactions for which are generally obtained by means of a funicular polygon, but they may be found by using substituted members in the truss as shown in Fig. 296. Set down the load line 1 to 6, Fig. 297, then 3-*c*, 4-*c* gives point *c*; *c*-*b*, 2-*b*, for point *b*; *b*-*d*, 5-*d*, for point *d*; *d*-*a*, 1-*a*, 6-*a*, for point *a*, which is also point 7. The reaction 1-7, 6-7, being found, the remainder of the stress diagram may be completed in the usual way.

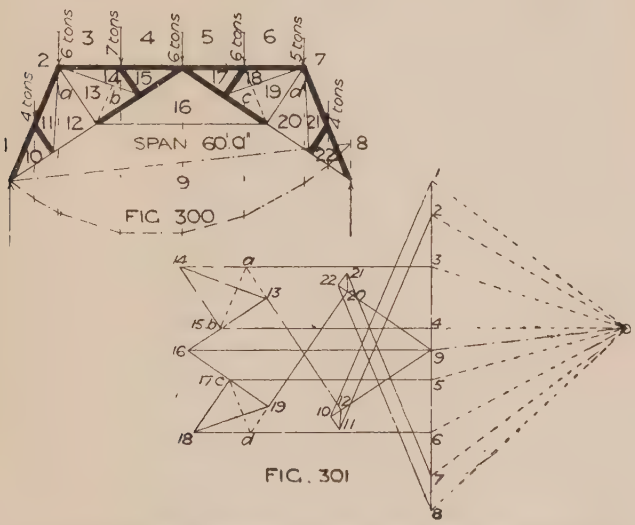


FIG. 300.—Frame diagram for large Roof Truss with flat top.
FIG. 301.—Stress diagram for ditto.

Fig. 298 shows a truss with a lantern light, the stresses in which require the aid of substituted members as shown by dotted lines. Fig. 299 is the corresponding stress diagram.

Fig. 300 shows a roof truss of rather curious form submitted to the author some time ago. The truss was 60 ft. span. Adopting substituted members, the stress diagram will be worked as shown in Fig. 301.

Fig. 302 shows a large compound roof truss for a span of 75 ft., with vertical loading. Substituted members are inserted to give the spaces *O P Q R* when the stress diagram Fig. 303 can be constructed without difficulty.

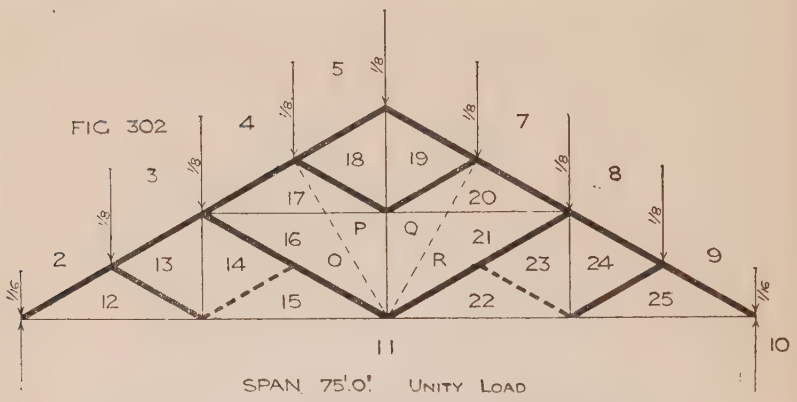


FIG. 302.—Frame diagram for Truss 75-ft. span.

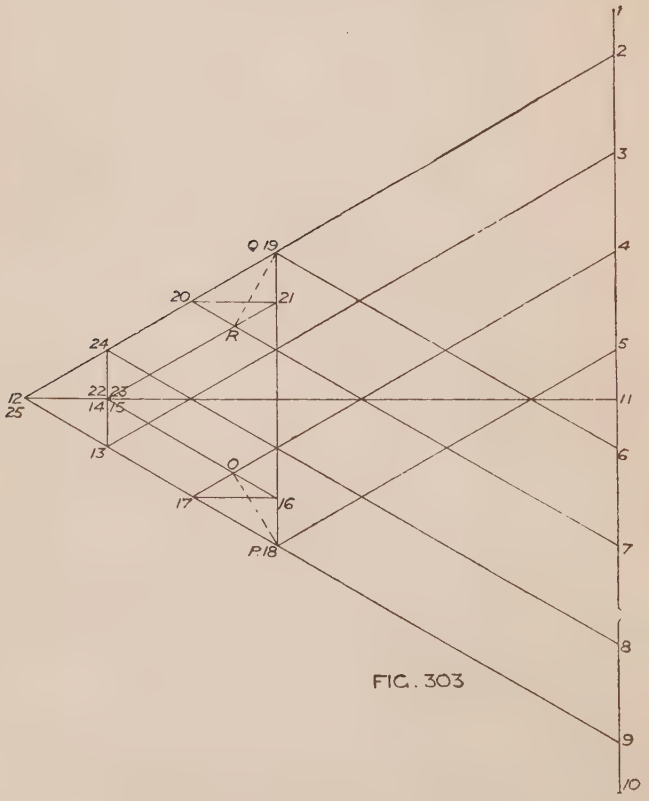


FIG. 303.—Stress diagram for Truss 75-ft. span.

These examples will be sufficient to show the general method of dealing with structures where the stresses are indeterminate by the ordinary means of reciprocal diagrams. It will be seen that the general principle is to ignore the actual trussing in the first instance, drawing substituted members from the various points of application of the loads to a joint on the lower side of the truss, and then to revert to the actual trussing when the stress in the intervening member is obtained.

Some girders with compound trussing, such as a Whipple-Murphy girder, cannot, however, be dealt with in this way, and require special treatment, which will be described in the next chapter.

CHAPTER XXVI

Various Types of Braced Girders—Example of Finding Stresses in Whipple-Murphy Girder by Analysis of Component Frames

Braced girders are of many forms, of which the Warren girder, lattice girder, lattice girder with verticals, and trellis girder have already been described; these have a uniform arrangement of bars in every bay. Besides these there are a few other parallel flange girders with irregular bracing. The

FIG. 304 — *Pratt Truss*

FIG. 305 — *Modified Pratt Truss*

FIG. 306 — *Howe Truss*

Pratt truss or N girder, Fig. 304, with vertical struts and inclined ties, was widely used on early railways in America. It was constructed with timber for the booms or flanges, floor and struts, and wrought iron for the diagonal ties; the centre bays had usually an additional tie to prevent distortion of the bridge when partially loaded. The modified Pratt truss, Fig. 305, had the top boom continued down to the base at each end, the last verticals then being in tension. The Howe truss, Fig. 306, had inclined struts with vertical ties and extra ties at the central

bays. When iron was substituted for timber, owing to the number of bridges burnt down by sparks from the locomotive engines, the same types of girder were used, but the Howe truss has been perhaps the design most often adopted both in America

FIG. 307 — *Brunel Girder*

FIG. 308 — *Bowstring Girder*

FIG. 309 — *Fish-bellied Girder*

FIG. 310 — *Hog-backed Girder*

FIG. 311 — *Fink Truss*

FIG. 312 — *Bollman Truss*

and elsewhere. In England when plate web girders fell into disuse for large spans the Howe truss was adopted, and this has been succeeded by the Whipple-Murphy girder, or Linville truss, Fig. 322, also an importation from America. As the stresses in this form of truss give some special difficulties, an example will

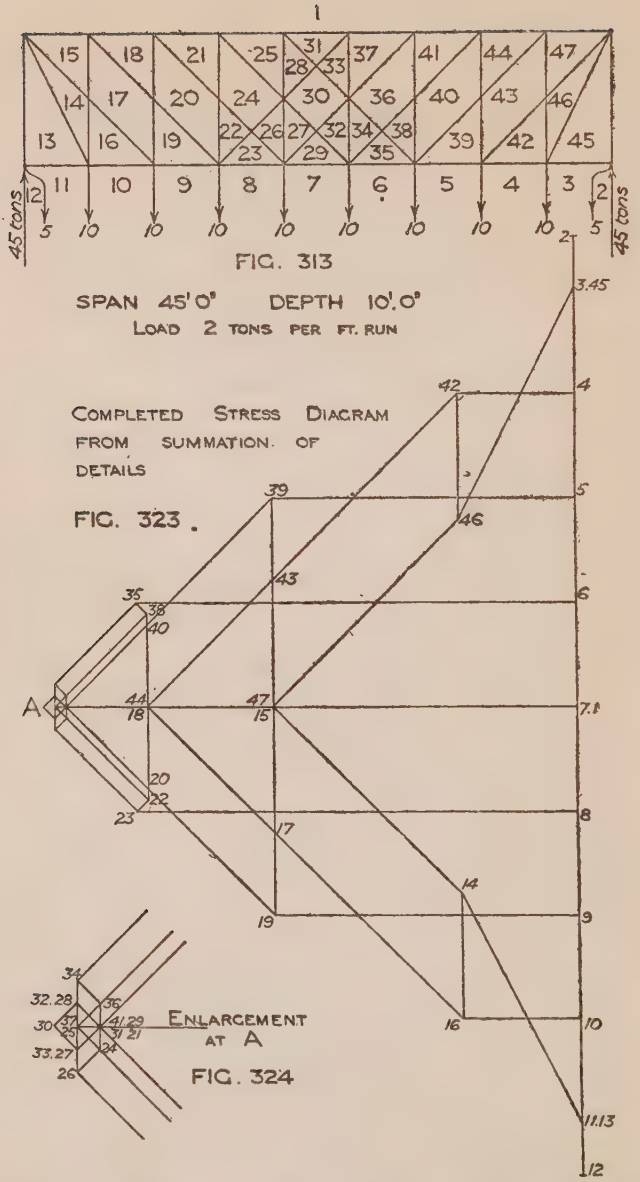


FIG. 313.—Frame diagram of Whipple-Murphy Girder, 45-ft. span, with Loads. FIG. 323.—Complete Stress diagram for ditto. FIG. 324.—Enlargement at A in Fig. 323.

be taken, but it may be as well first to call attention to a few other forms of braced girders. Fig. 307 shows a Brunel girder,

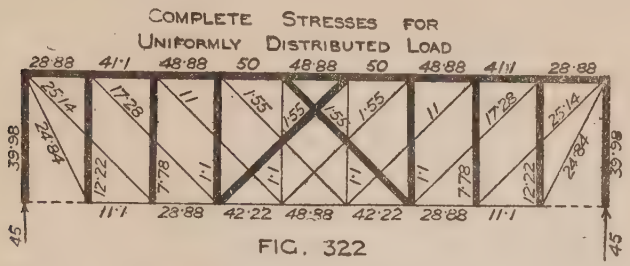


FIG. 322.—Frame diagram with complete Stresses marked on.

where the depth varies as the ordinates to a parabola, giving uniform stress throughout the top and bottom flanges; Fig. 308, a bowstring or bow and string girder where the top flange only is

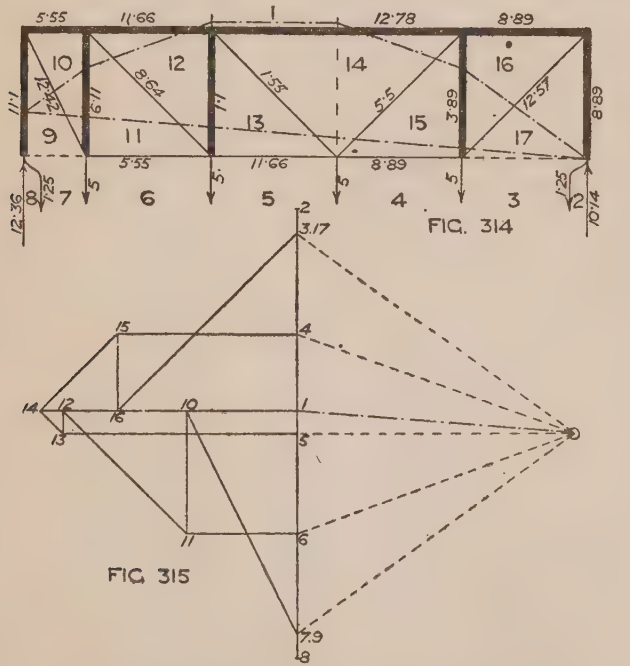


FIG. 314.—Part frame diagram of Whipple-Murphy Girder.
FIG. 315.—Stress diagram for ditto.

parabolic, and Fig. 309, which shows an inverted bowstring or fish-bellied girder. In these the web members may be all equally inclined, or alternately vertical and inclined, or crossing each other; in fact, any form of lattice may be adopted so long as the outlines of the top and bottom members remain unaltered. The fish-bellied girder is the usual type for travelling cranes

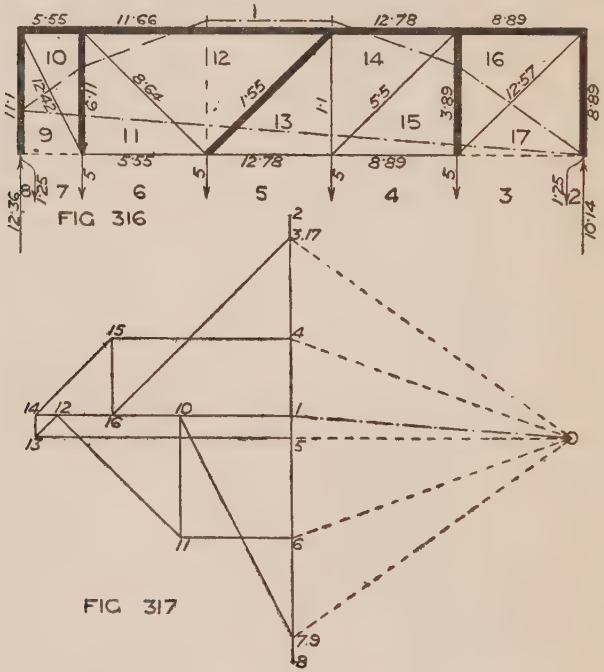


Fig. 316.—Part frame diagram of Girder.
Fig. 317.—Stress diagram for ditto.

when the top flange is required to be level for the crab to run upon. Sometimes the bowstring girder is terminated with vertical ends, as Fig. 310; it then becomes known as a hog-backed girder, and may be looked upon as a Pratt truss reduced in depth towards the ends. Another distinct set of types is furnished by trussed beams, of which the Fink truss, Fig. 311, and the Bollman truss, Fig. 312, are special forms.

As an example of the stress diagrams for a Whipple-Murphy

girder, the skeleton elevation Fig. 313 may be taken. This is purposely made unusually short in order to keep the diagram compact, but it is sufficient to show the mode of working by a detailed graphic analysis of the stresses. Generally there would be more bays, up to, say, sixteen, the two end ones having the top flange brought down diagonally to the abutments, as in the

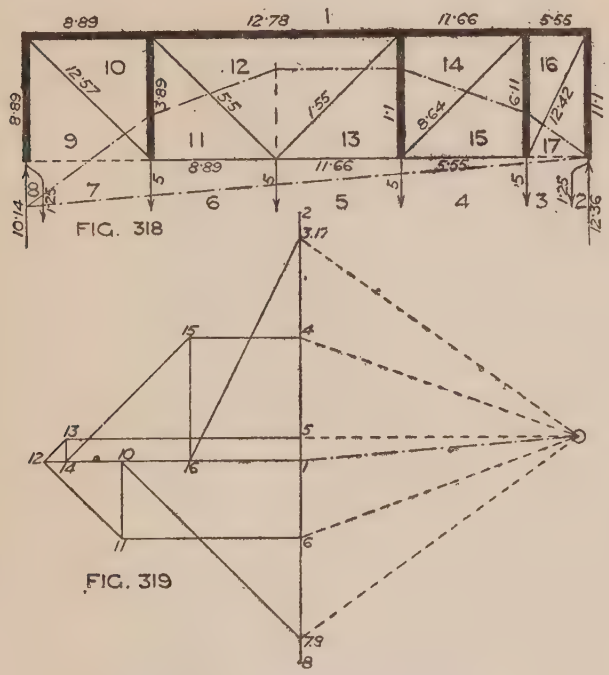


Fig. 318.—Part frame diagram of Girder.
Fig. 319.—Stress diagram for ditto.

modified Pratt truss. The span taken in the example is 45 ft., the depth 10 ft., and the load two tons per foot run. The girder must be assumed to consist of several component girders superposed, the loads being divided equally between the superposed vertical members. Commence with Fig. 314, which has the steep diagonal, and draw the vertical and inclined members as they follow on. Take half the load at each apex that is shown, and construct the stress diagram. To do this the load line must

first be drawn, and point 1, giving the reactions, will be obtained from the funicular polygon in the usual way, and the stress diagram Fig. 315 may then be constructed without difficulty. Next take Fig. 316 with the steep diagonal in the first bay again, and follow with the bracing members as shown; the stress diagram Fig. 317 will be constructed as before, and the stresses

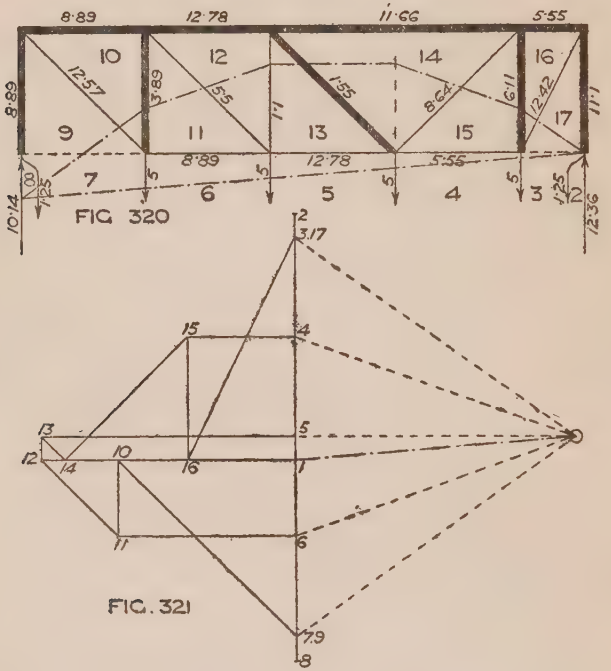


FIG. 320.—Part frame diagram of Girder.
FIG. 321.—Stress diagram for ditto.

should be scaled off, and marked upon the frame diagrams. Now take the 45 deg. diagonal crossing the first two bays, and follow with the bracing, as shown in Fig. 318, and draw the stress diagram Fig. 319. The remaining component truss will be as Fig. 320, and the stress diagram as Fig. 321.

Having now obtained the stresses in all the bars of the component girders, they may be summed up and inserted on Fig. 322, which shows the complete stresses on the whole girder

for a uniformly-distributed load. It is interesting to note that although these stresses could not be determined by a single stress diagram, it is possible to construct such a diagram from the known stresses, as in Fig. 323, and it is useful to draw this as a check upon the work. The stresses at the extreme point of the diagram are so small that this part has been enlarged in Fig. 324. In some cases when a complete diagram is constructed from component stresses, the method of constructing it direct may be discovered upon inspection, but this does not appear to be one of those cases. By a similar method to that shown above, the stresses due to a rolling load may be determined, but it would require something like sixteen additional diagrams, allowing for the load to advance consecutively to each junction on the bottom flange.

In the actual construction of the girder the top flange is usually of box girder shape with an open bottom, and the bottom flange is composed of two groups of plates placed vertically with the web members between. An arched portal frequently connects the top flanges of the girder at each end, and two or more overhead stay girders are placed at intermediate points.

CHAPTER XXVII

Bowstring Lattice Girder Footbridge—Complete Calculations and Design

A DESIGN will now be made for a bowstring lattice girder footbridge 60-ft. span, with a clear width between girders of 7 ft. 6 ins.

Weight of men in crowds . . . . 1 cwt. per ft. sup.
„ allowed on public buildings . . = 1½ cwts. „ „
„ „ on warehouse floors . . = 2½ cwts. „ „

For this bridge, say 1½ cwts. per foot super. Assume width of girder flanges 9 ins., then the girders will be 8 ft. 3 ins. centre to centre.

Total net load on bridge $\frac{60 \times \text{say } 8 \times 1{\cdot}5}{20} =$ . . 36 tons.

Weight of structure, say $\frac{WL}{200} = \frac{36 \times 100}{200} =$ . . 18 tons.

Gross load on each girder $\frac{36 + 18}{2} =$. 27 tons distributed.

Centre depth of girder, say $\frac{1}{10}$ span $= \frac{60}{10} =$ . . 6 ft.

Say 10 bays with vertical bars and bracing, each . 6 ft. wide.

Load on each apex $= \frac{27}{10} =$ . . . . . 2·7 tons.

Frame diagram of girder, see Fig. 325.
Reciprocal stress diagram, see Fig. 326.
Scale off stresses and insert on Fig. 325.

Size of rivets, say . . . . . ¾ in.
Single shear value, each . . . . . 2·2 tons.
End compression bars . . . . . 10·8 tons.
Unsupported length, say . . . . . 86 ins.
Number of rivets each end $\frac{10{\cdot}8}{2{\cdot}2} =$ say . . . 5 rivets.

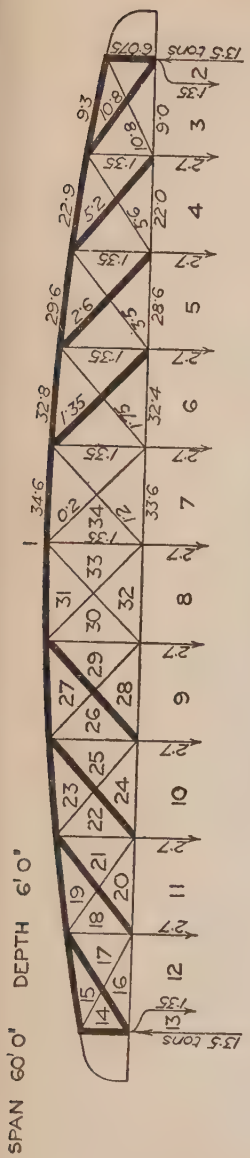


Fig. 325.—Frame diagram of Girder with Loads.

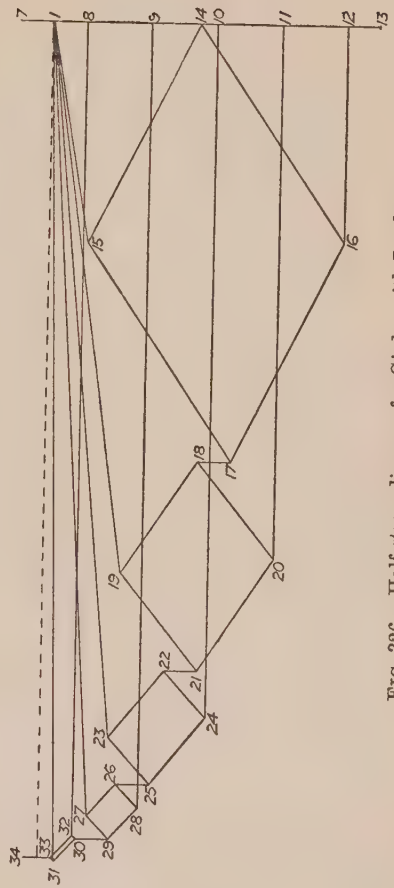


Fig. 326.—Half stress diagram for Girder with Loads.

Try a $3\frac{1}{2}$ by $3\frac{1}{2}$ by $\frac{7}{16}$ angle, taken as one end fixed, one rounded.

Then $W = \dfrac{fS}{1 + \dfrac{m}{nq}\left(\dfrac{l}{d}\right)^2} = \dfrac{6{\cdot}5 \times 2{\cdot}9}{1 + \dfrac{2{\cdot}5}{\frac{1}{2} \times 3{,}750}\left(\dfrac{86}{3{\cdot}5}\right)^2} = 10{\cdot}41$ tons.

Next compression bar . . . . . 5·2 tons.
Unsupported length, say . . . . . 92 ins.
Number of rivets each end $\dfrac{5{\cdot}2}{2{\cdot}2}$, say . . . . 3 rivets.

Try a 3 by 3 by $\frac{3}{8}$ angle.

Then safe load $W = \dfrac{6{\cdot}5 \times 2{\cdot}11}{1 + \dfrac{2{\cdot}5}{\frac{1}{2} \times 3{,}750}\left(\dfrac{92}{3}\right)^2} = .$. 6·1 tons.

Next compression bar . . . . . 2·6 tons.
Unsupported length, say . . . . . 97 ins.
Number of rivets each end $\dfrac{2{\cdot}6}{2{\cdot}2} =$ say . . . 1 rivet.

Try a $2\frac{1}{2}$ by $2\frac{1}{2}$ by $\frac{5}{16}$ angle.

Then safe load $W = \dfrac{6{\cdot}5 \times 1{\cdot}47}{1 + \dfrac{2{\cdot}5}{\frac{1}{2} \times 3{,}750}\left(\dfrac{97}{2{\cdot}5}\right)^2} = .$. 3·18 tons.

Note.—These are good proportions for the angles and it is not possible to provide them of exactly the right strength. The remainder may be similar to the last one found, as it is useless to go below a workmanlike size.

Width of bar required for a given number of rivets will be found by allowing $1\frac{1}{2}$ d from centre of outer rivet to edge of plate, and $2\frac{1}{2}$ d centre to centre of rivets. Diameter of rivet head, say $1\frac{2}{3}$ d.

A tension bar with one $\frac{3}{4}$-in. rivet in end must therefore be not less than 3 d wide $= 3 \times \frac{3}{4} = 2\frac{1}{4}$ in. wide, giving a net width of 2 $d = 2 \times \frac{3}{4} =$. $1\frac{1}{2}$ in.

A bar with two rivets in the same line will be $5\frac{1}{2}$ d wide $= 5\frac{1}{2} \times \frac{3}{4} = 4{\cdot}125$ ins. wide, say 4 ins. giving a net width of $4 - 2 \times \frac{3}{4}$ . . . 2·5 ins.

End tension bar . . . . . 10·2 tons.
Number of rivets $\dfrac{10{\cdot}2}{2{\cdot}2} = 4{\cdot}6$, say . . . . 5 rivets.

Net width, say 2·5 ins. thickness $\frac{10{\cdot}2}{2{\cdot}5 \times 6{\cdot}5} = {\cdot}627$ in.

Practical dimensions, say . . . . . 4 ins. by $\frac{5}{8}$ in.

Next tension bar . . . . . 5·6 tons.

Number of rivets $\frac{5{\cdot}6}{2{\cdot}2} = 2{\cdot}5$, say . . . . 3 rivets.

Try a bar 3 ins. wide then net width = $2\frac{1}{4}$ ins.,

thickness $\frac{5{\cdot}6}{2{\cdot}25 \times 6{\cdot}5} = {\cdot}38$ in.

Practical dimensions with rivets placed singly . 3 ins. by $\frac{3}{8}$ in.

Next tension bar . . . . . 3·5 tons.

Number of rivets $\frac{3{\cdot}5}{2{\cdot}2} = 1{\cdot}6$, say . . . . 2 rivets.

Try a bar $2\frac{1}{2}$ ins. wide, thickness $\frac{3{\cdot}5}{1{\cdot}75 \times 6{\cdot}5} = {\cdot}3$ ins.

Practical dimensions, say . . . . . $2\frac{1}{2}$ ins. by $\frac{3}{8}$ in.

The other bars may be the same size with one rivet in each end.

End upright, compression . . . . 6·075 tons.

As this is supported throughout its length and will be strengthened by a web plate, a $2\frac{1}{2}$-in. by $2\frac{1}{2}$-in. by $\frac{5}{16}$-in. angle will be sufficient.

The other uprights are all in tension with equal stresses of 1·35 tons, and may be made the same as the smallest diagonal tension bars, $2\frac{1}{2}$ ins. by $\frac{3}{8}$ in., with one rivet in each end.

Top flange.

A strut may be applied on outside of each main girder for support, $1\frac{1}{2}$ bays apart, the unsupported length of compression flange sideways will, therefore, be $1\frac{1}{2} \times 6 =$ . . . . . 9 ft.

Maximum stress in compression = ... 34·6 tons.

Try two 4-in. by 4-in. by $\frac{7}{16}$ angles, ends fixed.

Then $W = \dfrac{6{\cdot}5 \times 6{\cdot}6}{1 + \frac{1}{\frac{1}{2} \times 3{,}750}\left(\frac{9 \times 12}{8}\right)^2} =$ . . . 39·0 tons.

The unsupported length vertically will be 1 bay = . 6 ft.

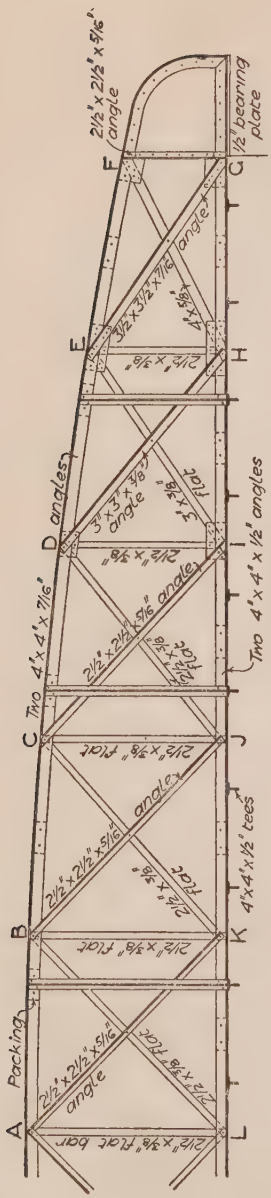


FIG. 327.—Half-elevation of Girder.

Then for the two 4-in. by 4-in. by $\frac{7}{16}$-in. angles, ends fixed—

$$W = \frac{6{\cdot}5 \times 6{\cdot}6}{1 + \frac{1}{\frac{1}{2} \times 3{,}750}\left(\frac{6 \times 12}{4}\right)^2} =$$

36·66 tons.

Bottom flange.

Maximum stress in tension = . . . 33·6 tons.

The sectional area of two 4-in. by 4-in. by $\frac{1}{2}$-in. angles, less one rivet hole = . . . 6·75 sq. ins.

In addition to the direct stress there will be the bending moment due to the load on the deck beams at centre of each bay. The load on each deck beam = 7·5 × 3 × 1·5 = 33·75 cwts., or 16·875 cwts. on each girder, which produces a bending moment in the centre of each bay of $\frac{16{\cdot}875 \times 1{\cdot}5 \times 12}{20} = 15{\cdot}18$ ton-ins.

The distance (x) of neutral axis from nearest edge of section =

$$\frac{4^2 \times \frac{1}{2} + \frac{1}{2}^2(4 - \frac{1}{2})}{2 \times 3{\cdot}75} = 1{\cdot}2 \text{ in.},$$

$y = 4 - 1{\cdot}2 = 2{\cdot}8$ ins. $I = \frac{1 \times 2{\cdot}8^3 + 8 \times 1{\cdot}2^3 - (8-1)(1{\cdot}2 - \frac{1}{2})^3}{3}$

$= 11{\cdot}125$, then $Z_x = \frac{I}{x} = \frac{11{\cdot}125}{1{\cdot}2} =$ 9·27, and $Z_y = \frac{I}{y} = \frac{11{\cdot}125}{2{\cdot}8} = 3{\cdot}97$.

Then $-\frac{W}{A} + \frac{M}{Z_y} = -\frac{33{\cdot}6}{6{\cdot}75} + \frac{15{\cdot}18}{3{\cdot}97}$

$= -5 + 3{\cdot}82 = -1{\cdot}18$ tons

per sq. in. tension and $-\frac{W}{A}-\frac{M}{Z_x}=-\frac{33{\cdot}6}{6{\cdot}75}-\frac{15{\cdot}18}{9{\cdot}27}=-5-1{\cdot}63=-6{\cdot}63$ tons per sq. in. tension, which is just safe.

The load on the deck beams will be $7{\cdot}5\times3\times1{\cdot}5=33{\cdot}75$ cwts. distributed, producing a bending moment of $\frac{WL}{8}=\frac{33{\cdot}75\times7{\cdot}5\times12}{20\times8}=18{\cdot}96$ ton-ins. Try a 4-in. by 4-in. by $\frac{1}{2}$-in. tee section with a moment of inertia of 5·56, then $x=\frac{4^2\times\frac{1}{2}+\frac{1}{2}^2(4-\frac{1}{2})}{2\times3{\cdot}75}=1{\cdot}2$ ins. and $y=4-1{\cdot}2=2{\cdot}8$ ins., $Z_x=\frac{5{\cdot}56}{1{\cdot}2}=4{\cdot}63$, and $Z_y=\frac{5{\cdot}56}{2{\cdot}8}=2{\cdot}0$. The stresses will then be

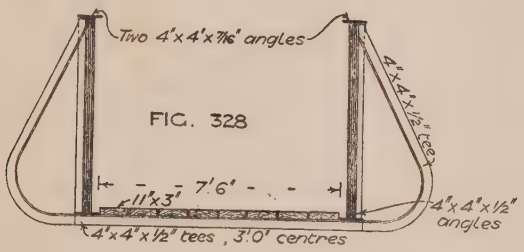


FIG. 328.—Section through centre of Footbridge.

$\frac{18{\cdot}96}{4{\cdot}63}=4{\cdot}09$ tons per sq. in. compression and $\frac{18{\cdot}96}{2{\cdot}0}=9{\cdot}48$ tons per sq. in. tension.

These tees may be continued round to form the outside struts as shown in Figs. 327 and 328, and they should be quite sufficient to stiffen the girders against the pressure of an ordinary crowd, but if a crowd of football spectators might at any time be collected against one girder, an overhead arched tie should connect it at the centre to the other girder.

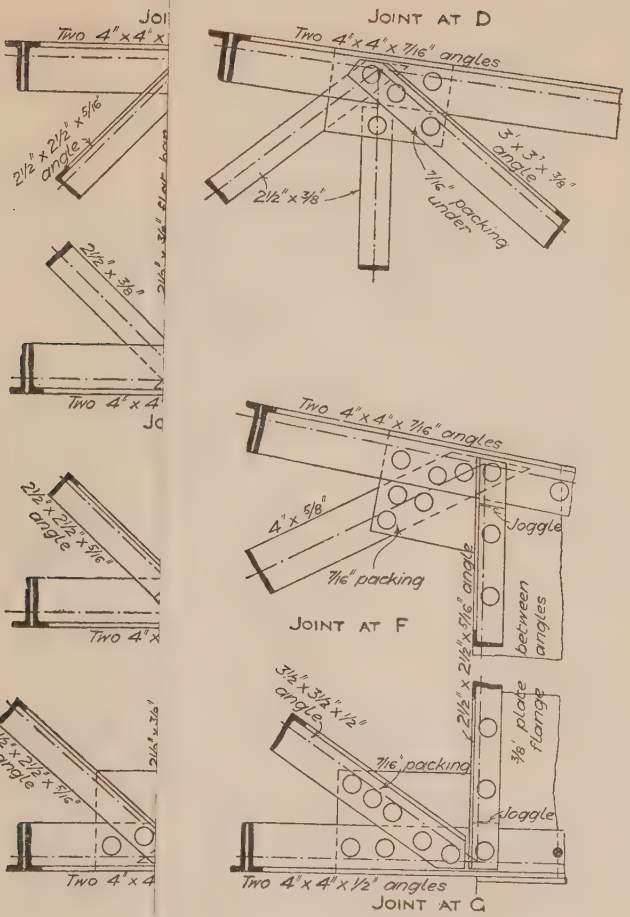
The floor boards will have to support a load of $6\times1\frac{1}{2}=9$ cwts.; assuming them to be 11 ins. wide, the thickness required $=\sqrt{\frac{6\times9}{11}}=\sqrt{5}=2{\cdot}236$, say $2\frac{1}{4}$ ins., which allows $\frac{3}{4}$ in. for wear on 11-in. by 3-in. planks.

Both flanges will, therefore, run through the same size, with

packing between the angles, 12 ins. long, with two rivets through each as shown in Fig. 327.

The joints may now be designed. They should be drawn out singly to a large scale as shown in outside elevation in Figs. 329 to 338, which are lettered to correspond with the letters on the elevation Fig. 327.

For wind bracing, 2-in. by $\frac{3}{8}$-in. bars may be riveted to the cross beams and checked into underside of flooring, passing diagonally over, say, three intermediate beams.


Two 4" x 4" x
2½" x 2½" x 5/16" angle
2½" x 3/8"
Two 4" x 4"
Joint at D
Two 4" x 4" x 7/16" angles
3" x 3" x 7/8" angle
2½" x 3/8"
7/16" packing under
Two 4" x 4" x 7/16" angles
4" x 5/8"
7/16" packing
Joggle
2½" x 2½" x 7/16" angle
between angles
Joint at F
2½" x 2½" x 5/16" angle
Two 4" x
3½" x 3½" x ½" angle
2½" x 2½" x 5/16"
7/16" packing
⅜" plate flange
Joggle
Two 4" x 4" x ½" angles
Joint at G

[*To face p.* 214.

CHAPTER XXVIII

Stresses in Hanging Ropes—Suspension Bridges—Various Types—Stiffened Roadways

Wire-rope suspension bridges are of elegant appearance, light and economical for foot-passengers up to perhaps 250 ft. or 300 ft. span and up to 10 ft. in width. The decking may be cambered or horizontal, the latter being suitable for very light vehicular traffic. For heavier loads and larger spans suspension bridges are not considered economical until a very considerable span is reached, one American writer putting the lower limit at 2,000 ft.; they are then composed of bars with pin ends or of bent girders for the suspending portion and of lattice girders for the suspended part.

When a chain or rope is suspended between two points and carries only its own weight it is uniformly loaded *throughout its length* and takes the form of a catenary curve. If it carried a load uniformly distributed over the horizontal span it would take the form of a parabola. In practice the weight of the platform, which may or may not be uniformly loaded, and the weight of rope are combined and the rope then takes the form of what is called a modified catenary. The catenary is indistinguishable from a parabola when its dip does not exceed $\frac{1}{10}$ span; when deeper the catenary is seen to be nearer to a circular arc. For practical purposes a parabola may always be drawn, as the weight of platform is invariably in excess of the weight of the rope. Taking W for the whole load, L the span and d the dip, the tension in the rope at centre $= \frac{WL}{8\,d}$, and the tension at the ends $= \sqrt{\left(\frac{WL}{8\,d}\right)^2 + \left(\frac{W}{2}\right)^2}$. It should be observed that the formula for central stress is the same as for a beam supported at the ends with a uniformly distributed load. Fig. 339 shows a graphic approximation to the curve for a versine or dip of

$\frac{1}{4}$ to $\frac{1}{8}$ of span. Let $A\,B\,C$, Fig. 340, represent the elevation of a suspension chain uniformly loaded throughout its length or

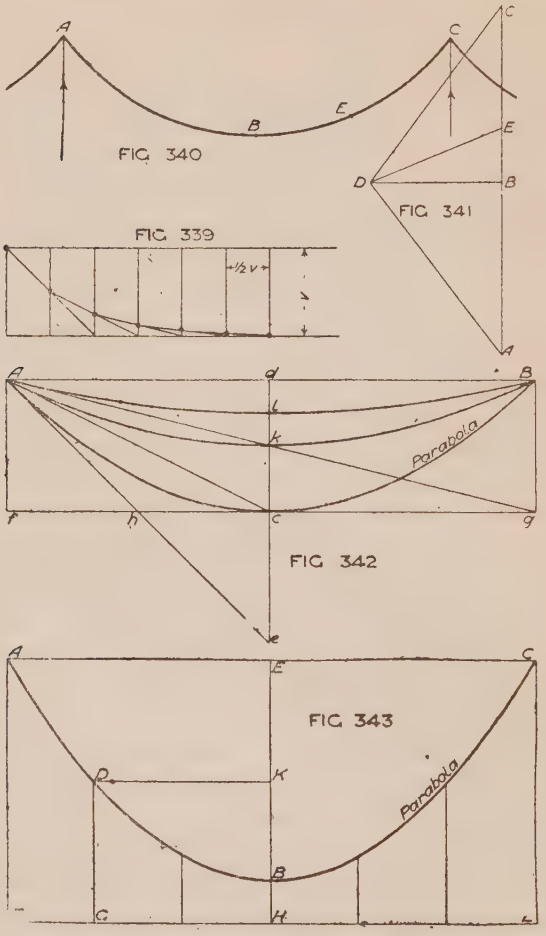


FIG. 339.—Graphic approximation to Catenary Curve. FIG. 340.—Elevation of Suspension Chain. FIG. 341.—Stress diagram for ditto. FIG. 342.—Diagram of comparative Stress with varying dip. FIG. 343.—Diagram for length of Suspension Rods.

throughout the span with a total load $A\,C$, then by graphic diagram, Fig. 341, let $A\,C$ = line of loads, draw $A\,D$, $C\,D$, parallel to tangents to chain at A and C, which will vary with the method

of loading, then the lengths $A\,D$, $C\,D$ will give the stress at points A and C. Similarly $D\,B$ parallel to tangent at B, or $D\,E$ parallel to tangent at E will give the stresses at those points.

Fig. 342 shows a graphic diagram of the comparative stress upon a suspended chain, from which it will be seen that half the dip gives double the horizontal stress. In this figure the length of the diagonal lines gives the comparative stresses at point A, and the distances $f\,h$, $f\,c$, and $f\,g$ give the horizontal stress at centre of dip for the parabolic curves passing through c, k, and l. The length of the suspension rods in a suspension bridge may be found by calculation, as shown in Fig. 343, where the length of any suspension rod as $D\,G$ at a distance $D\,K$ from the centre of span $= \dfrac{(DK)^2 \times EB}{(AE)^2} + BH$. The tension in the backstay of a cable depends in the first place upon the pull of the cable, and in the second place in certain circumstances upon the angle of the backstay. This can easily be determined by the parallelogram of forces; for example, assume a cable suspension bridge of 100-ft. span with a dip of 10 ft., each cable carrying a total distributed load of 150 tons, and free to move over the piers, the backstay making an angle of 45 degrees. The tension in the cable at the piers will be

$$= \sqrt{\left(\frac{WL}{8\,d}\right)^2 + \left(\frac{W}{2}\right)^2} = \sqrt{\left(\frac{150 \times 100}{8 \times 10}\right)^2 + \left(\frac{150}{2}\right)^2} =$$

$\sqrt{(187{\cdot}5)^2 + 75^2} = \sqrt{40781{\cdot}25} = 201{\cdot}9$ tons. The tension in the cable at the lowest extremity will be $\dfrac{WL}{8\,d} = \dfrac{150 \times 100}{8 \times 10} =$ 187·5 tons. As the cable is free to move over the piers the tension in the backstay will be the same as at top of cable = 201·9 tons, and the pressure on the piers by parallelogram of forces as in Fig. 344 will equal 224 tons.

If the cable had not been free to move over the pier the backstay would have had to resist, instead of the foregoing, the horizontal pull at top of pier of 187·5 tons, which is the same as the horizontal tension at lowest point of cable. The tension in the backstay would have been $\sqrt{2 \times (187{\cdot}5)^2} = 265{\cdot}1$ tons, as shown by Fig. 345. The pier would have $\dfrac{W}{2} = \dfrac{150}{2} = 75$ tons

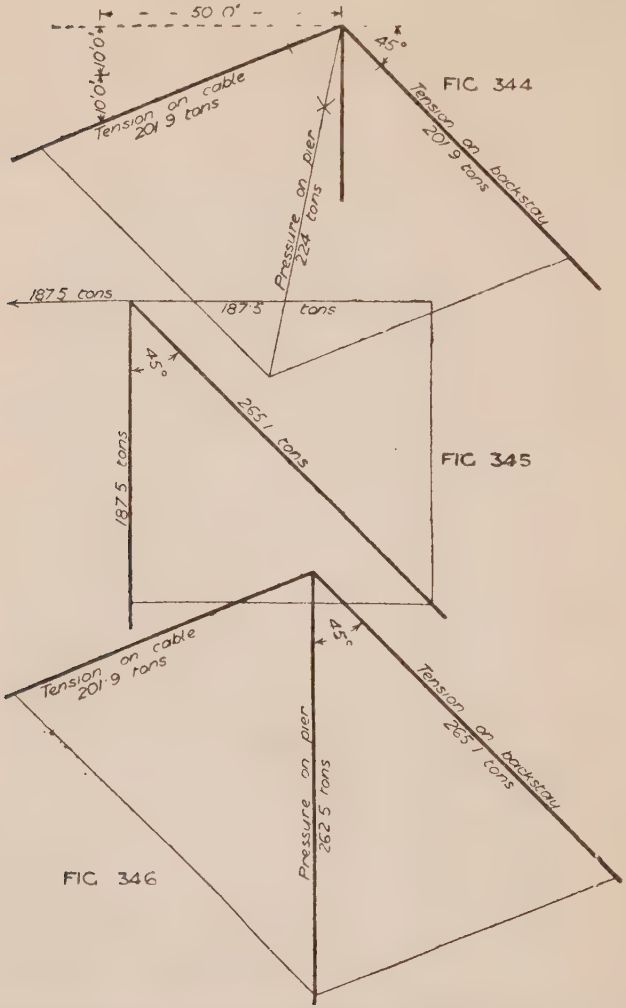


FIG. 344.—Parallelogram showing Stresses at support when Cable is free to move. FIG. 345.—Ditto, showing Stress in Backstay when Cable is fixed. FIG. 346.—Ditto, showing Stresses at support when Cable is fixed.

due to the pull of the cable, and 187·5 tons due to the pull of the backstay = 75 + 187·5 = 262·5 tons total, as shown by Fig. 346.

If one end of the chain be higher than the other it will hang in the shape of two semi-parabolas, and the tension at the lowest

point will be the same as if the whole chain were composed of two equal sides like either of the portions, but the lowest part will not be in the centre, so that the whole parabolas would not be of equal span.

For example, take Fig. 347. The tension at lowest point C of

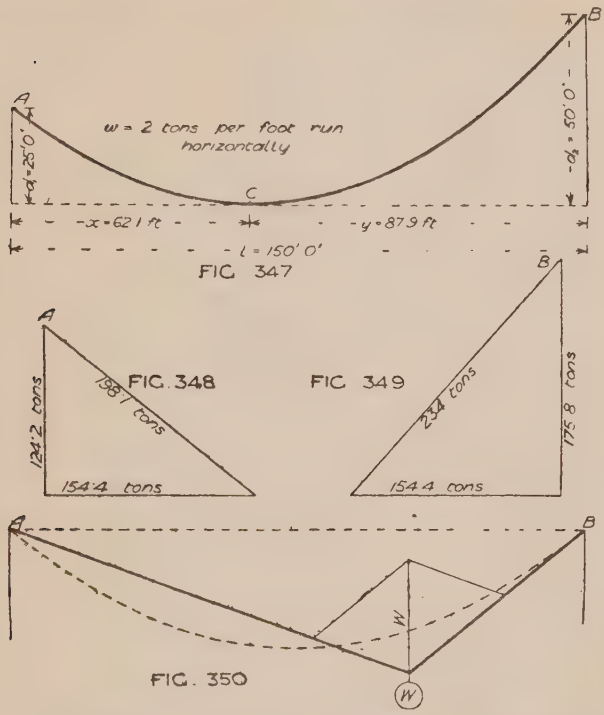


FIG 347

FIG. 348

FIG 349

FIG. 350

FIG. 347.—Elevation of Chain with one support higher than the other. FIG. 348.—Stresses at support A. FIG. 349.—Stresses at support B. FIG. 350.—Suspension chain with unsymmetrical loading.

chain $= \frac{wl^2}{8\,d} = \frac{w\,(2\,x)^2}{8\,d_1}$ which equals $\frac{w\,(2\,y)^2}{8\,d_2}$, therefore $\frac{wx^2}{2\,d_1} = \frac{w\,y^2}{2\,d_2}$, therefore also $\frac{x^2}{d_1} = \frac{y^2}{d_2}$, and the ratio $\frac{x^2}{y^2}$ = the ratio $\frac{d_1}{d_2}$ which is $\frac{25}{50}$ or $\frac{1}{2}$, whence $\frac{x}{y} = \frac{1}{1{\cdot}414}$, therefore $x = \frac{1}{1 + 1{\cdot}414} \times 150 =$ 62·1 ft., and $y = \frac{1{\cdot}414}{1{\cdot}414 + 1} \times 150 = 87{\cdot}9$ ft., and $62{\cdot}1 + 87{\cdot}9 =$

150 ft. total span. Taking the left-hand portion the tension at $C = \frac{wx^2}{2\,d_1} = \frac{2\,(62{\cdot}1)^2}{2 \times 25} = 154{\cdot}27$ tons, and from the right-hand

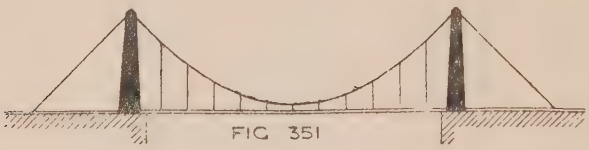


FIG. 351

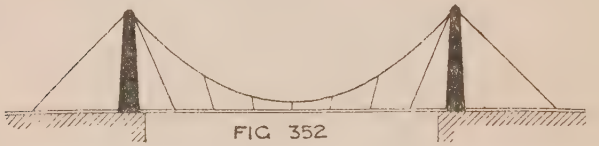


FIG. 352

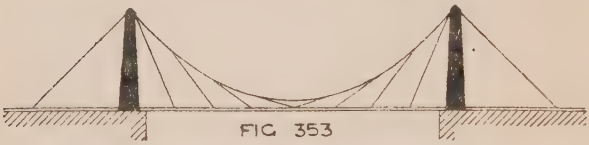


FIG. 353

FIG. 354

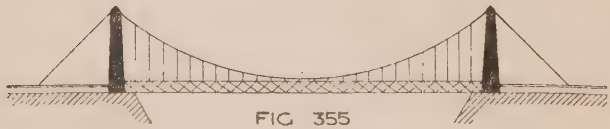


FIG. 355

FIG. 351.—Unstiffened Suspension Bridge. FIGS. 352 and 353.—Suspension Bridges with simple sloping Suspension Links. FIG. 354.—Suspension Bridge with braced Suspension Links. FIG. 355.—Ditto, with vertical Suspension Links and Lattice Girder.

portion the tension at $C = \frac{wy^2}{2\,d_2} = \frac{2\,(87{\cdot}9)^2}{2 \times 50} = 154{\cdot}52$ tons, the small difference being due to the omission of further decimals in x and y. The mean may be taken as 154·4 tons. The tension in the chain at support A will be the direct pull in the chain due to the load, but may be resolved into two forces, a horizontal

pull and a vertical pressure; the former will be the tension just found, 154·4 tons, and the latter will be the load upon the length $x = wx = 2 \times 62{\cdot}1 = 124{\cdot}2$ tons. Then the tension at A will

FIG. 356

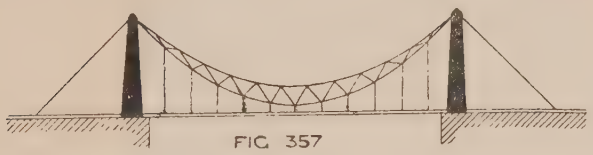


FIG. 357

FIG. 358

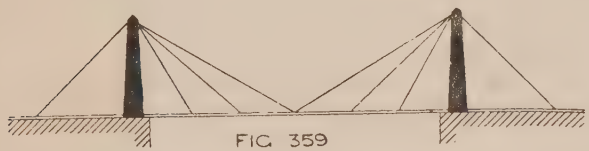


FIG. 359

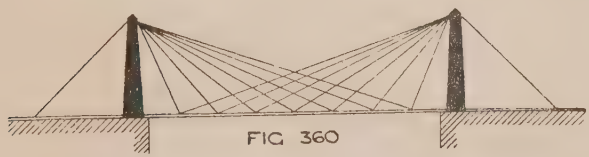


FIG. 360

FIG. 356.—Suspension Bridge with Girders in two pairs hinged at centre. FIG. 357.—Ditto, with two Suspension Chains braced together. FIG. 358.—Ditto, with Chains in two segments braced together. FIG. 359.—Ordish Suspension Bridge. FIG. 360.—Bollman Truss Bridge.

be the resultant $= \sqrt{(154{\cdot}4)^2 + (124{\cdot}2)^2} = \sqrt{39{,}265} = 198{\cdot}1$ tons, and the triangle of forces will be as Fig. 348. The tension in the chain at support B will be in the same manner $= \sqrt{(154{\cdot}4)^2 + (175{\cdot}8)^2} = \sqrt{54{,}745} = 234$ tons, and the triangle of forces as Fig. 349

The difficulty that occurs with a simple roadway suspended from a hanging chain or rope is that unequal loading alters the curve of the chain, and tends to pull it into the position shown by the full lines from A and B in Fig. 350 where the dotted line shows the ordinary parabola produced by a uniformly distributed load. The stresses at A and B will be the parts cut off from the straight lines by the parallelogram of forces as drawn. To overcome this difficulty various devices have been proposed or carried out as shown in the following figures: Fig. 351 shows an ordinary unstiffened suspension bridge where the suspension rods are vertical and the flooring not braced by side girders, so that a concentrated load on any part tends to draw the curved chain downwards at that part. Fig. 352 and Fig. 353 show modifications of the system of simple sloping suspension links by which the roadway is partially stiffened, but the tension in the lower member is increased. Fig. 354 shows a bridge with the vertical suspension link braced so as to form a lattice girder of varying depth and prevent any tendency to distortion under a travelling concentrated load. Fig. 355 shows vertical suspension links and the roadway stiffened by a lattice girder on each side. In this arrangement an increase of temperature causes the main chain to sag and deflect the girder with greatly increased stresses due to the deflection. Fig. 356 shows a similar arrangement but with the lattice girder in two pairs hinged in the centre and free to move on roller bearings at the ends. In this case the stresses due to deflection are avoided, but a heavy bridge is the result. In the bridge shown by Fig. 357 the chains are stiffened by bracing two together so that a light roadway only is required. In Fig. 358 the chains are braced in two segments hinged at their junction. This is considered to be a very good arrangement. Fig. 359 shows an Ordish suspension bridge with a series of straight chains forming a simple arrangement for small bridges. Fig. 360 shows a Bollman truss. In this arrangement the roadway is carried by direct chains to the two piers from several different points, preventing the wave motion of the single chain under a rolling load, but introducing great stresses upon change of temperature from the very unequal length of the chains.

CHAPTER XXIX

Theory of Arches—Stability of Voussoirs—Curve of Thrust—Rolling Loads

Every distribution of loading upon an arch gives a certain "curve of thrust" or "linear arch" which is the theoretical centre line of the required arch ring. With a constant distribution of loading there would be no difficulty in doing this, making the arch ring just thick enough to withstand the direct thrust, but with travelling loads there would be a certain range in the shape of the curve which would necessitate a corresponding increase in the thickness of arch ring. In practice it is not convenient to shape the arch according to the theoretical outline, and frequently architectural considerations outweigh everything else, so that all the engineer can do is to provide sufficient thickness in order that the working limit of stress shall not be exceeded. The curve of thrust for any loading is the exact equivalent to a suspended chain with the same loading, but inverted. In finding the curve of thrust it is not necessary to work from the actual voussoirs; any arbitrary divisions may be taken, either radial or vertical.

If we consider a single voussoir under ideal conditions, the forces acting upon it are three in number, they act through the centre of gravity, and are in equilibrium, as shown in Fig. 361. When the line of thrust cuts the centre of a joint the pressure is equally distributed over the joint, but should it be out of the centre, but not outside the middle third, the pressure will be greater than the average on the side where the thrust is nearer to the edge, and less than the average on the other side. If it passes through the edge of middle third, the pressure at the edge on the narrow side will be double the average, and the pressure at the opposite edge will be reduced to nothing. It is commonly supposed that if the line of thrust passes anywhere

through the middle third of every joint the arch will be safe, but this is a false assumption, as the actual pressure may be beyond the safe limit of stress. All that can truly be said is that in these circumstances there will be no tension. When the joints are cemented, or in the case of a homogeneous arch as of steel, or concrete, or reinforced concrete, a certain amount of tension may be permissible, then the line of thrust may pass outside

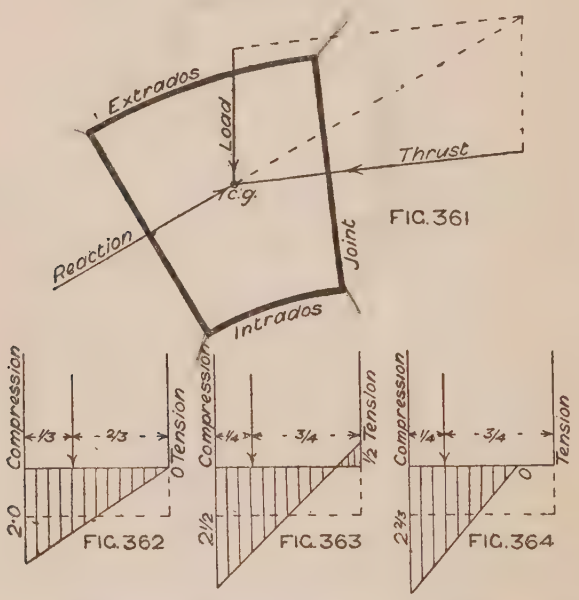


FIG. 361.—Diagram of Forces acting on voussoir. FIGS. 362 to 364.—Distribution of Stress over Joint according to position of Resultant.

the middle third, the calculation of the stress at the outer edges being made by the formula $\frac{W}{A} \pm \frac{M}{Z}$. There is another theory, due to Professor Crofton, of the Royal Military Academy, Woolwich, by which the joint is not necessarily subject to tension, although the line of thrust passes outside the middle third. Upon this theory part of the joint is subject to no stress, and the maximum compression is increased. The three cases are shown in Figs. 362, 363, and 364, with the comparative values of the

stress produced by the same load, the dotted parallelogram showing the average pressure.

There is one other condition to note with regard to the joints of an arch, and that is that the line of thrust must not make a greater angle with the joint than the limiting angle of resistance, which is the angle θ in $\mu = \tan \theta$, μ being the coefficient of friction. If this angle is exceeded, slipping will take place.

An example will now be taken of a symmetrically-loaded arch (Fig. 365). Having drawn the elevation of the arch to any convenient scale, a vertical line may be set up at each end, and then on the left of the centre line a horizontal line may be drawn for the roadway at such height that the weight of the filling material will be represented to the same scale as the weight of the arch ring, or, in other words, if the arch ring of stone weighs 140 lbs. per cubic foot and the material above weighs 90 lbs. per cubic foot, the height of the roadway will be reduced in the ratio of $\frac{90}{140}$. In the same way another height will be set off above the roadway to represent the external load to the same scale. Then any area measured on the face will represent the actual load on that part of the arch as if it were all solid stone. Take first the left-hand side of the arch and load up to the centre line. Prick it through on to a piece of drawing-paper, cut it out, suspend from two points, and draw vertical lines by a set square to find the centre of gravity at the point of intersection. Find the area of the outline by planimeter, and from this the weight. Say it equals 13,500 lbs. Draw a horizontal line from, say, the upper edge of the middle third of the arch ring at the crown to meet the centre of gravity line, and from the point where it cuts set off the load *W*. From the same point draw a line through the lower edge of middle third of arch ring at the skewback, and complete the parallelogram. Then scale off the horizontal and inclined sides, when the thrust at crown will be found to be 15,600 lbs. and the thrust at skewback 20,500 lbs. These are called the primary thrusts.

On the other half of the arch the arch ring and load may be divided up into any number of parts, equal or unequal, but preferably into parts of equal width by a series of parallel lines.

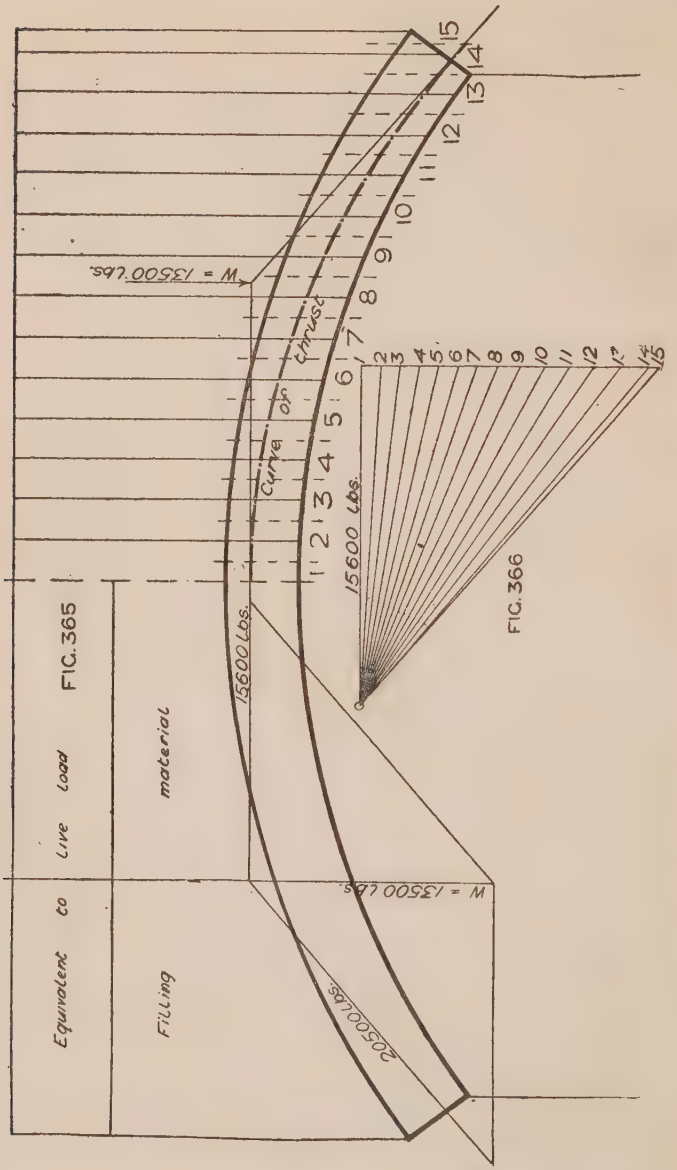
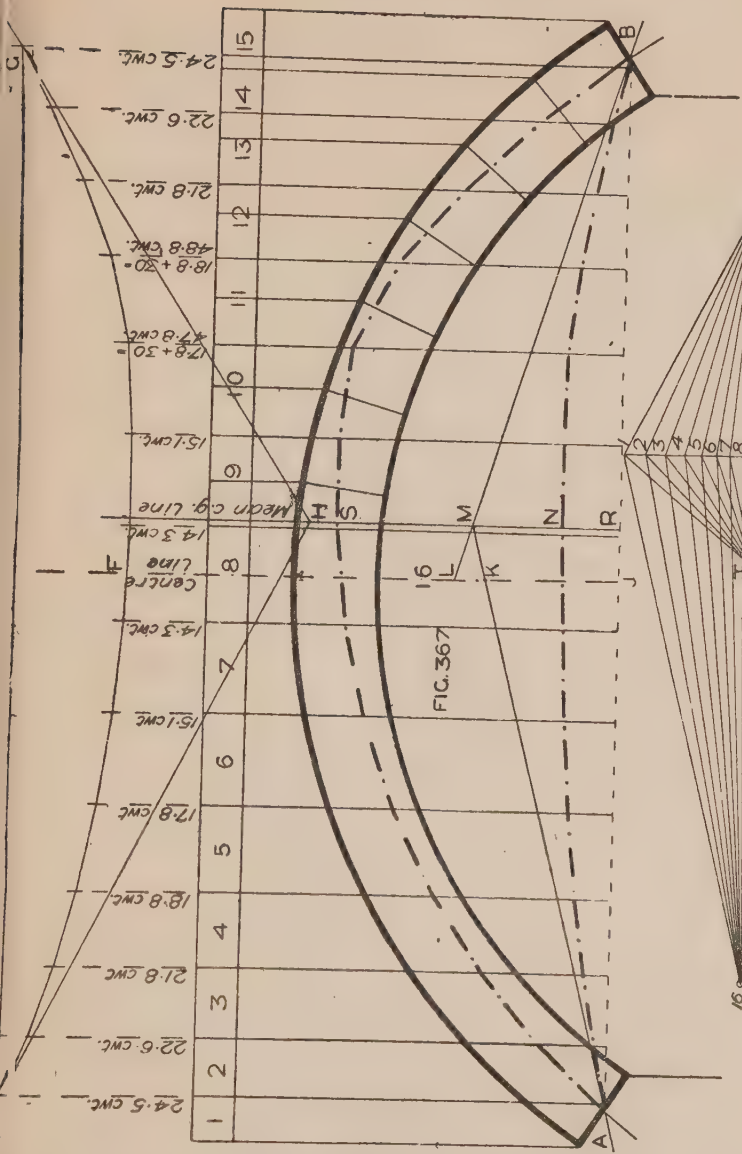


FIG. 365.—Elevation of Arch with Curve of Thrust for symmetrical Loading.
FIG. 366.—Vector diagram for ditto.

The loads, being the weight of each division, are then scaled off by taking the height through the centre of each division, and marked down on the vertical load line in Fig. 366. The



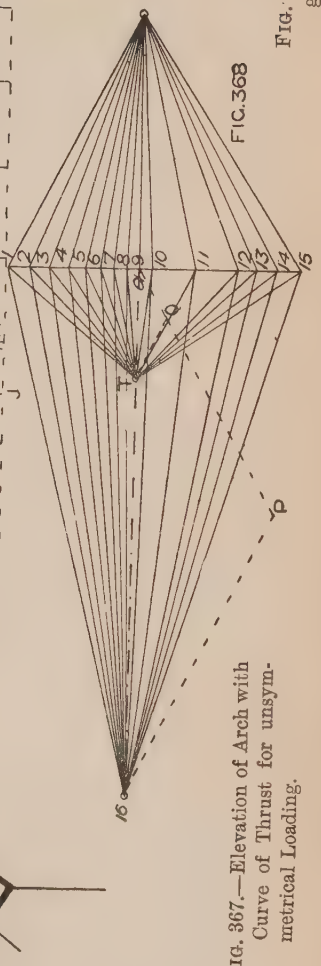


FIG. 368.—Vector diagram for ditto.

FIG. 367.—Elevation of Arch with Curve of Thrust for unsymmetrical Loading.

horizontal thrust at crown of arch is marked from the top of the load line and the inclined line to close the figure should then be exactly equal to the inclined thrust at skewback. The next step

will be to draw vectors from the points on the load line to the top angle or pole of the diagram, and parallel with each of these draw the portions of the line of thrust on the arch across the respective divisions. If the two figures are marked with corresponding numbers upon Bow's system of notation, the work will be simplified and the risk of error reduced. This mode of working will show that the line of thrust for the whole arch is really a funicular polygon, and will follow the same laws, one of which is that by altering the position of the ends or the vertical scale we may modify the shape or course of the line without altering its character.

There are various possible lines of thrust for the same loading, according to the position taken for the two points through which the primary thrusts pass. Each point may be taken anywhere within the range of the middle third of the depth, or even slightly beyond; but Nature chooses that line of thrust which will, on the whole, give the minimum stresses, and although we may endeavour to find it by trial we cannot always succeed. At best we can only hope for an approximation to accuracy which shall be sufficient for practical purposes.

Most architects, and perhaps some other people, are under the impression that a semi-circular arch produces no thrust, as the skewbacks are horizontal, but this is incorrect. There will in all cases be a horizontal thrust at the skewback equal to the thrust at the crown, and this is indicated by the linear arch or curve of thrust meeting the springing line of the arch in an inclined direction.

Sometimes, particularly with semi-circular and elliptical arches, the line of thrust will fall below the intrados of the arch at the haunches, indicating a considerable compression at the intrados and tension at the extrados: so much so that some cases of failure that have been brought to the notice of the writer have been traced to this cause. According to the conditions of loading there will be a certain minimum thickness of arch ring necessary to keep the line of thrust within it, but it must be remembered that while the line of thrust is the theoretical form which the arch should take, practically it is only the instrument by which the distribution of stress in the actual arch is ascertained.

The line of thrust can be modified in shape by the distribution of the load, a concentrated load causing an upward bulge in the curve at the point of application of the load. It is this fact which creates a difficulty in the case of a travelling load, the bulge following the load in the same way that the crest of a wave travels across a sheet of water. The critical point in the stresses occurs when a concentrated travelling load is over one of the haunches. A traction-engine passing over an arched bridge perhaps gives rise to the greatest variation of stress. The transmission of the load from the surface of the ground to the extrados of the arch may be considered as spreading it in a cone shape at an angle of 60 deg. from the line of contact of the wheels. It is usual also to allow for a further live load of say $1\frac{1}{4}$ cwts. per foot super. distributed over the whole surface. Fig. 367 shows an arch with structural load, external distributed live load, and concentrated live load, or rolling load taken at the worst point—*i.e.*, over one of the haunches. Draw out the arch and loads, and divide up into sections. Mark the weight on the centre of gravity line of each part, and number the spaces. Then draw the load line (Fig. 368), select a pole, draw vectors, and parallel with these draw the funicular polygon *E F G H*; the intersection *H* will give the position of the line through the mean centre of gravity of all the loads. From the pole 0 draw 0-*a* parallel to *E G*, giving the vertical reactions at the abutments 1-*a* and 15-*a*. Next from the centre of the skewbacks draw a horizontal line *A B* cutting the centre line of the arch in *J*. From *J* set up *J K* equal to 1-*a*, and *J L* equal to 15-*a*. Join *A K* and *B L*, and produce *A K* to meet *B L* at point *M*, which should be on the centre of gravity line through *H*. Then, in Fig. 368, draw 1-16 parallel to *A M* and 15-16 parallel to *B M* to give point 16. Lines are now drawn from the remaining points on the load line to 16, and the curve of thrust *A N B* constructed parallel to these lines. This does not pass through the arch at all; but, nevertheless, it is a true thrust curve, and only wants raising into its position by magnifying all its vertical ordinates, or bringing point 16 sufficiently near to the load line to increase the angle of all the vectors, so that the line of thrust shall pass through the centre of arch ring at point *S*. To do this set off a

line a-P (Fig. 368) at any angle, and make a-P equal to height from R on line $A\ B$ to centre of arch ring S and join 16-P. On a-P set off a-Q equal to height from R to N, and from Q draw a line $Q\ T$ parallel with 16-P to meet a-16. Then T is the pole required, from which vectors may be drawn and the curve of thrust $A\ S\ B$ constructed as before. This curve appears to lie fairly equal within the width of the arch ring, but the true test will be to work out the maximum stresses at the worst points, say 5 and 11. The thrust would be measured from Fig. 368 and the leverage for bending moment, or distance from centre of arch ring from Fig. 367, using the formula $\frac{W}{A} \pm \frac{M}{Z}$ for the stresses incurred, which should not exceed the safe load on the material.

CHAPTER XXX

Stability of Retaining Walls—Details of Design—Surcharged Retaining Walls

The stability of walls is a very important branch of structural mechanics. The forces to be resisted are usually the pressure of the wind or the pressure of earth or water. It has been already shown that the wind pressure varies with the height from the ground, the friction of the earth's surface and the projections upon it decreasing the effective pressure very considerably. An ordinary boundary wall is seldom subjected to a pressure exceeding about 20 lbs. to a square foot, and the safe thickness in such a case for brickwork in mortar may be taken as one-ninth of the height. The walls of dwelling-houses and warehouses may be proportioned by Rondelet's rule, upon which the London Building Act was based. This rule is $T = \frac{cLH}{\sqrt{L^2 + H^2}}$ where $T =$ thickness of wall in feet, $H =$ height in feet, $L =$ unsupported length in feet, $c =$ constant $= \frac{1}{12}$ for warehouses and $\frac{1}{20}$ for dwelling-houses. The usual footings will be required in all cases, viz., as many courses as there are half bricks ($4\frac{1}{2}$ ins.) in the thickness of the wall, each course projecting $2\frac{1}{4}$ ins. on each side of the wall, and the width of the bottom course being twice the thickness of the wall. The concrete below the footings will usually project 6 ins. beyond the footings on each side, and be not less than 9 ins. deep, but the projection may be equal to half the thickness of the wall with a depth equal to twice the projection.

There are various theories of the pressure of earth against retaining walls, but that of Rankine is usually adopted, and so far his assumptions seem to be well founded, for no failures have been traced to any inadequacy of the formula. He assumes that the earth is a granular mass, and that the line of rupture, or

plane of separation upon a sudden slipping, lies half-way between the angle of repose of the earth and a vertical line, the portion between the vertical and the line of rupture forming a wedge

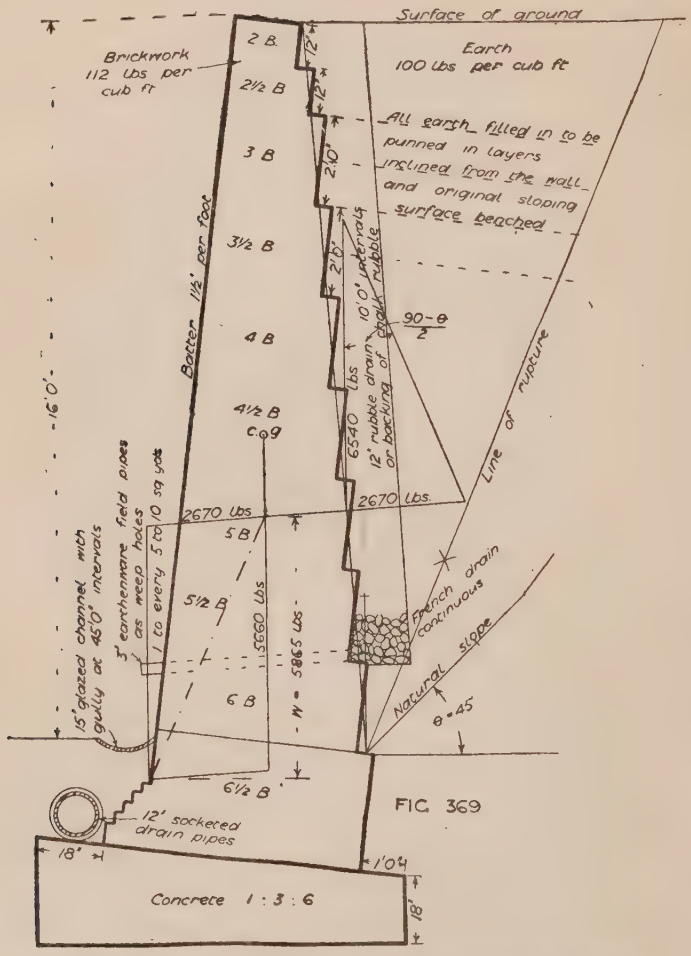


FIG. 369

FIG. 369.—Section of Brick Retaining Wall to support earth at back.

that tends to force the wall over by its weight and form. Considerable economy is produced by giving a batter to the face of the wall, by reason of the centre of gravity being thrown further back, and if the wall leaned back to the line of rupture there

would be no pressure upon it, and it would not need to be more than a thin protective skin to the earth.

Although it would be possible to frame a formula for the requisite thickness with a given allowance for batter and natural slope of earth, it is usual and more convenient to draw a probable section, and then find graphically or calculate its stability under the given conditions. Fig. 369 represents the section of a brick retaining wall, with a batter of $1\frac{1}{2}$ ins. per foot, for a railway cutting. The courses are always laid at right angles to the batter, and the joint of least stability will, therefore, be that at the level of the lowest set-off on the back of the wall. Draw an equalising line through the set-offs, and from the lowest point draw a horizontal line and set up the natural slope of the earth according to the knowledge of the soil, say $\theta = 45$ deg. Then draw a short vertical line and bisect the space between this and the natural slope to obtain the line of rupture. It may be assumed that the line of rupture indicates the probable surface that would be left immediately upon the overturning of the wall, while the natural slope or angle of repose shows the surface it would weather down to in course of time. At one-third of the height of the wall, measuring from the joint of least stability, draw a line perpendicular to the equalising line to give the position and direction of the line of thrust. From their intersection set up a vertical line equal on any convenient scale to the weight of 1 ft. run of the whole wedge of earth between the line of rupture and the back of wall, and from the top set off an angle equal to the angle between the line of rupture and the vertical $\left(=\frac{90-\theta}{2} \text{ or } 45-\theta/2\right)$ to cut the line of thrust, giving the length from the wall to point of cut-off for the measure of the thrust. This mode of working may be different from that in use by the reader, but the advantage of it is that it applies equally whether the back of the wall be inclined inward, or outward, or vertical. The centre of gravity of the wall must now be found by cutting out a section and suspending it from two points. Then the weight of 1-ft. run of the wall must be calculated, a vertical line drawn through the centre of gravity of the wall, and the weight of wall measured downwards from the

intersection with the line of thrust produced. A parallelogram is then drawn with the thrust for one side, and the weight of wall for the adjacent side and the diagonal represents the resultant. The distance of the point where the resultant cuts the plane of least stability must be carefully measured from the centre of the joint. In this case it is 1·95 ft. and the vertical component of the resultant is 5,865 lbs. Then by the formula $\frac{W}{A} \pm \frac{M}{Z} =$

$\frac{5{,}865}{4{\cdot}5 \times 1} \pm \frac{5{,}865 \times 1{\cdot}95}{\frac{1}{6} \times 1 \times 4{\cdot}5^2} = 1{,}303 \pm 3{,}389 = +\ 4{,}692$ lbs., or 2·09 tons per sq. ft. compression and — 2,086 lbs., or 0·93 ton per sq. ft. tension, which will be quite safe, although the resultant cuts well beyond the middle third of the base.

A precisely similar method may be followed in ascertaining the stability of walls under water pressure. As water has an angle of repose or natural slope of 0 degrees the line of rupture will be at 45 deg. from the horizontal, and the thrust may be calculated from the weight of the wedge of water above it. Or, alternatively, the water pressure at any height being $w\ h$, where w = weight in 1 lb. per cubic foot and h = height or depth from surface in feet, the total pressure will be $\frac{1}{2}\ w\ h^2$ acting at one-third the height, against a vertical face, or $\frac{1}{2}\ w\ h^2$ cosec θ when the back of the wall slopes at an angle θ from the horizontal.

When a retaining wall supports a bank of earth higher than itself, it is said to be surcharged, and the thrust is greatly increased. Let Fig. 370 represent a surcharged wall of mass concrete weighing 120 lbs. per cubic ft., and supporting a bank of earth rising 10 ft. above top of wall at a slope of $2\frac{1}{2}$ to 1, the backing having a natural slope of 30 deg. The method of working will be as follows: Let $A\ B\ C\ D$ be the section of wall, then from $D\ C$ set-off $D\ E$ making the angle $C\ D\ E$ equal to the natural slope, and produce the slope of the surcharge to meet $D\ E$ at E. Bisect $D\ E$ in F and on $D\ E$ describe a semi-circle $E\ G\ D$, and from C draw a line at 90 deg. with $E\ D$ to cut the semi-circle in G. With radius $E\ G$ describe the arc $G\ H$ cutting $D\ E$ in H. Then the horizontal thrust at $\frac{1}{3}$ the height of wall will be $\frac{1}{2}\ w\ (D\ H)^2 = \frac{1}{2} \times 100 \times (13{\cdot}5)^2 = 9{,}112$ lbs.,

and the thrust parallel with the slope of surcharge will be found by diagram as shown = 9,800 lbs. The weight of wall $= \frac{6{\cdot}5 + 4}{2} \times 20 \times 120 = 12{,}600$ lbs., and the two forces may

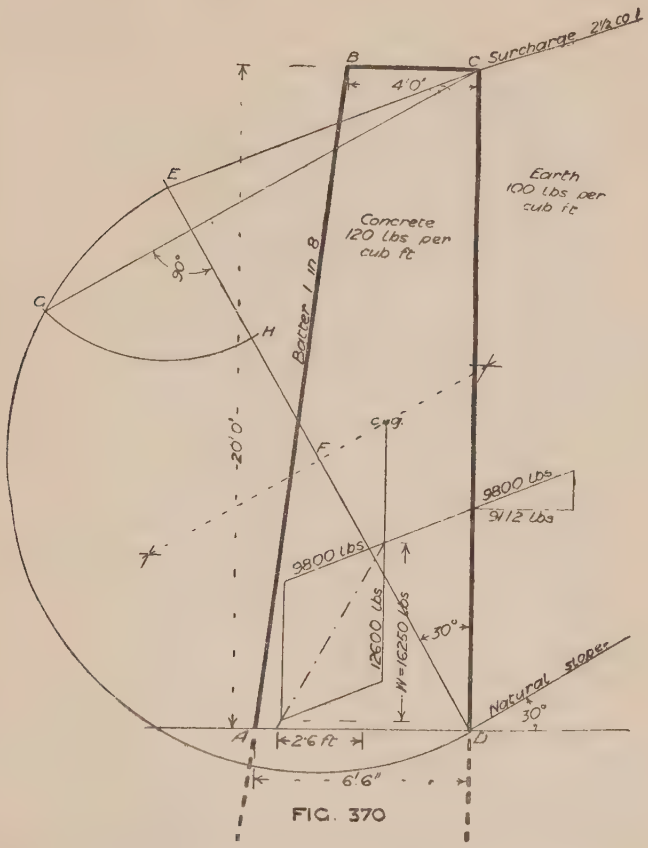


FIG. 370.—Section of Concrete Retaining Wall to support surcharged earth at back.

be combined, giving a resultant which cuts the base at 2·6 ft. from its centre. The thrust will not be affected by the height of the surcharge after the line of rupture is reached, but should it not rise so far the actual thrust will be proportionately reduced from what is given by this method of working. Let p = intensity

of pressure in lbs. per sq. ft., W = vertical component of resultant, A = sectional area of base, M = bending moment due to eccentricity of resultant, Z = section modulus of base. The stresses will then be $\frac{W}{A} \pm \frac{M}{Z} = \frac{16{,}250}{6{\cdot}5} \pm \frac{16{,}250 \times 2{\cdot}6}{\frac{1}{6}(1 \times 6{\cdot}5^2)} =$ 2,500 ± 6,000 = 8,500 lbs. or 3·8 tons per sq. ft. compression, and 3,500 lbs. or 1·56 tons per sq. ft. tension, which would be the natural condition of stress, and the concrete ought to be capable of easily sustaining this amount of tension, but if the concrete is assumed to have no strength in tension the compression on the outer edge will be increased and the calculation will become $p = \frac{2}{3}\frac{W}{d}$, where p and W are as before and d is distance of resultant *from edge of base*. Then $p = \frac{2}{3}\frac{W}{d} = \frac{2}{3} \times \frac{16{\cdot}250}{3{\cdot}25—2{\cdot}6}$ = 16,667 lbs. = 7·44 tons per sq. ft. compression at outer edge and nil at inner edge.

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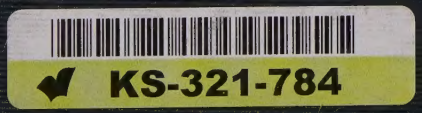
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